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TYPE-2 TAKAGI-SUGENO-KANG FUZZY LOGIC SYSTEM AND  
UNCERTAINTY IN MACHINING

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DÉPARTEMENT DE GÉNIE MÉCANIQUE  
ÉCOLE POLYTECHNIQUE DE MONTRÉAL

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Cette thèse intitulée

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MACHINING

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*So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality.*

– Albert Einstein

*As complexity rises, precise statements lose meaning and meaningful statements lose precision.*

– Lotfi Zadeh

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## RÉSUMÉ

L'objectif principal de cette thèse réside en l'application de l'algorithme d'identification TSK de type-2, basé sur la technique de regroupement soustractif, à des problèmes d'estimation de l'incertitude en usinage. Ce type d'applications a tendance à causer une augmentation de la dimensionnalité et afin d'y remédier, un système utilisant la logique floue de type-2 à ordre supérieur a été développé et implémenté. Cette thèse aborde une étude théorique sur la logique floue de Type 2 ainsi qu'une étude pratique sur l'estimation de la dynamique de l'usinage à haute vitesse.

La thèse est composée de trois articles publiés dans des journaux scientifiques. Les articles traitent de thématiques suivantes : le système TSK de logique floue de type 1, de type 2 et de type 2 augmenté, ainsi que la technique de regroupement soustractif. Le tout est appliqué à des séries de données temporelles du signal acoustique obtenu lors de l'usinage à haute vitesse.

Dans le cadre des avancements théoriques, cette thèse propose un algorithme d'identification TSK de type-2 basé sur la technique de regroupement soustractif qui a été appliqué avec succès à l'estimation de l'incertitude en usinage. Plus particulièrement, la modélisation des émissions acoustiques en usinage de précision a été utilisée afin de déterminer les différences en performance entre le TSK de type 2 et son équivalent de type 1. L'estimation des incertitudes avec la logique floue de type 2 offre une approche simple et efficace d'obtenir une solution exacte sans avoir besoin de comprendre la physique exacte du processus d'usinage en cours. De plus, cela permet d'évaluer les incertitudes associées à la prédiction causées par les erreurs de fabrication et le traitement de signal. De plus, il est possible d'établir une estimation de la condition de l'outil d'usinage en utilisant le logique floue de Type-2, car il existe une corrélation entre l'incertitude associée aux modèles développés et l'usure de l'outil.

Par ailleurs, une généralisation du système de logique floue TSK de type 2 ainsi qu'un système de logique floue TSK d'ordre supérieur—incluant son architecture, le moteur d'inférence et la méthode de design—ont été développés. Une modélisation du signal acoustique expérimental en tournage a été effectuée en utilisant un système de logique floue de type 2 à intervalles de second ordre. Ceci a été utilisé pour des fins de démontrer que ce genre de

systèmes peut non-seulement gérer efficacement les incertitudes au sein même du système de logique floue, mais est aussi capable de réduire le nombre de règles floues nécessaires à identifier le même système que celui d'un premier ordre.

La logique floue de Type-2 représente un développement récent et une nouvelle direction de recherche dans ce domaine. Cette thèse constitue un nouvel apport sur les points de vues théoriques et pratiques.

## ABSTRACT

The main objective of this thesis is to apply the type-2 TSK identification algorithm based on subtractive clustering method to real problem in uncertainty estimation in machining, and develop a high order type-2 fuzzy system to solve the problem of dimensionality. This thesis includes the theoretical studies on type-2 TSK FLS and experimental studies on application of type-2 FLS on uncertainty estimation of dynamics in high precision machining process.

This thesis is composed of three papers published in international journals. They cover the following topics: type-1 TSK FLS, first order interval type-2 FLS and high order Interval FLS modelling based on one experimental data set. The data set is a time series AE signal voltage for a high precision turning process.

For the theoretical contributions, this thesis introduces the development of fuzzy logic sets and systems, especially, TSK FLS – from type-1 TSK FLS to type-2 TSK FLS including subtractive clustering based type-2 identification algorithm. To solve the curse of dimensionality in type-2 interval systems, the generalized type-2 TSK FLS and its architecture, inference engine and design approach are proposed.

For the experimental studies, the same data sets recorded from the physical machining system is used to identify the type-1, first-order type-2 and high order type-2 fuzzy systems. The fuzzy models of the experimental results are compared to each other in order to show the differences between type-2 TSK system and its type-1 counterpart and demonstrate that type-2 modeling performs better and that the high order interval type-2 TSK FLS has the capability to overcome the problem of dimensionality AE. The results obtained in this thesis show that the type-2 fuzzy estimation not only provides a simpler way to arrive at a definite conclusion without understanding the exact physics of the machining process, but also assesses the uncertainties associated with the prediction caused by the manufacturing errors and signal processing. It is possible to establish a reliable type-2 fuzzy tool condition estimation method based on information of uncertainty in AE signal because AE uncertainty scheme corresponds to the complex tool wear state development. A second order IT2 TSK FLS has less rules than that of first order FLS, it can modeling the AE signal with similar RMSE. That prove that high order interval type-2 TSK FLS has the capability to overcome the problem of dimensionality.



Type-2 fuzzy logic is a new research direction and there exists no applications to machining. The main contributions of this thesis are applying the type-2 TSK identification algorithm based on subtractive clustering method on uncertainty estimation in dynamic in machining processes and proposing the generalized interval type-2 FLS which establishes the theoretical basics for high order type-2 FLS.

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## LIST OF ABBREVIATIONS AND SYMBOLES

AE	Acoustic emission
FBF	Fuzzy Basis Function
FL	Fuzzy Logic
FLS	Fuzzy Logic System
LSE	Least Square Error
MF	Membership Function
MIMO	Multi-Input Multi-Output
MISO	Multi-Input Single-Output
RMSE	Root Mean Square Error
SF	Signal feature
TCM	Tool condition monitoring
TSK	Takagi-Sugeno-Kang
$a_j^k$	Spread percentage of the cluster center
$b_j^k$	Spread percentage of fuzzy numbers
$c_j^k$	Centre (mean) of fuzzy numbers
$f$	Mathematical function
$f^k$	Total firing strengths interval sets for the $k$ th rule in a type-2 TSK FLS
$k$	Centre (mean) of fuzzy numbers
$m$	Number of rules
$n$	Number of antecedents
$P_i$	Potential value of data vector

$p_j^k$	Regression parameters for $j$ th antecedent in $k$ th TSK rule $r_k$
$\tilde{p}_j^k$	Fuzzy consequent parameters for $j$ th antecedent in $k$ th TSK rule $r_k$ in a type-2 TSK FLS
$Q_{jk}$	Type-1 fuzzy sets on universe of discourse for $j$ th antecedent in the $k$ th TSK rule.
$\tilde{Q}_{jk}$	Type-2 fuzzy sets on universe of discourse for $j$ th antecedent in the $k$ th TSK rule
$r$	Number of consequents
$r_a$	Hypersphere <i>cluster radius</i> in data space
$r_b$	Hypersphere <i>penalty radius</i> in data space
$r_k$	$k$ th rule, $k \in [1, m]$
$s_j^k$	Spread of fuzzy number,
$sumMu_k$	Total membership grade for the $k$ th rule
$X_k$	Universe of discourses for the $k$ th rule
$w$	Number of data points
$w^*$	Crisp output
$\tilde{w}^k$	Interval value of the consequent of the $k$ th rule in a type-2 TSK FLS
$\tilde{w}$	Interval set of total output for all rules of the type-2 TSK FLS
$w_k^*$	Conclusion for $k$ th TSK rule $r_k$
$W_{im}$	Fuzzy model output
$W_{is}$	System output
$x_k$	$k$ th linguistic variable, $k \in [1, n]$
$x^i$	$i$ th normalized data vector $i \in [1, w]$
$x_k^0$	$k$ th crisp input, $k \in [1, n]$
$x_i^j$	$j$ th element in the vector $x^i$ , $j \in [1, n]$
$x_{jk}^*$	Cluster center
$Z$	Linguistic variables

$\alpha_k$	Rule firing strength (weight) for $k$ th type-1 TSK rule $r_k$
$\beta_k$	Variable
$\sigma$	Deviation of Gaussian MFs
$\sigma_j^k$	Deviation of $j$ th Gaussian MF in $k$ th rule
$\mu_{jk}(x)$	Membership grade for $j$ th antecedent in $k$ th TSK rule $r_k$ , $x \in X$
$\tilde{\mu}_{jk}$	Interval membership grade for $j$ th antecedent in $k$ th TSK rule $r_k$ ,
$\eta$	Squash factor
$\varepsilon_-$	Reject ratio
$\varepsilon_+$	Accept ratio

## CHAPTER 1 INTRODUCTION

In this chapter, basic terminologies of *Fuzzy logic* (FL), type-1 and type-2 *Fuzzy logic systems* (FLS), and subtractive clustering method are introduced. Moreover, our previous study is recalled.

### 1.1 Fuzzy Logic System

FL was introduced by Zadeh in the 1960's (Zadeh, 1965). FL is a superset of conventional (Boolean) logic that has been extended to process data by allowing partial set membership rather than crisp set membership or non-membership. FL is a problem-solving control system methodology. FL provides a simple way to arrive at a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information.

A FLS is an expert system that uses a collection of fuzzy membership functions (MFs) and rules to reason about data. If-then rule statements are used to formulate the conditional statements that comprise FL. The rules in a fuzzy expert system are usually of a form similar to the following:

$$\begin{array}{l} \text{IF } x_1 \text{ is } Q_1 \text{ and } x_2 \text{ is } Q_2 \\ \text{THEN } Z \text{ is } w \end{array}$$

where  $x_1$  and  $x_2$  are linguistic input variables,  $Z$  is a linguistic output variable (a name for a data value to be computed).  $Q_1$  and  $Q_2$  are linguistic values defined by fuzzy sets on the universes of discourse  $X_1$  and  $X_2$ . The IF-part of the rule “ $x_1$  is  $Q_1$  and  $x_2$  is  $Q_2$ ” is called the *antecedent* or premise, while the THEN-part of the rule “ $Z$  is  $w$ ” is called the *consequent* or conclusion. The consequent  $w$  can be a fuzzy set or a mathematical function.

A FLS that is described completely in terms of type-1 fuzzy sets is called a *type-1 FLS*, whereas a FLS that is described using at least one type-2 fuzzy set is called a *type-2 FLS*.

Type-1 FLSs cannot directly handle rule uncertainties because they use type-1 fuzzy sets that are certain. Type-2 FLSs, on the other hand, are very useful in circumstances in which it is

difficult to determine an exact membership function for a fuzzy set; hence, they can be used to handle rule uncertainties and even measurement uncertainties.

Type-2 FLSs have been developed that satisfy the following fundamental design requirement (Mendel, 2001):

When all sources of uncertainty disappear, a type-2 FLS must reduce to a comparable type-1 FLS.

Takagi-Sugeno-Kang type fuzzy model structure, also being referred to as TSK fuzzy logic systems (FLSs) (Takagi & Sugeno, 1985; Sugeno & Kang, 1986), after Takagi, Sugeno & Kang, was proposed in an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and mathematical function in the consequent part. Usually, conclusion function is in form of dynamic linear equation. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviours of the system in those regions.

The identification of TSK fuzzy system using clustering involves formation of clusters in the data space and translation of these clusters into TSK rules such that the model obtained is close to the system to be identified. Subtractive clustering method (Chiu, 1994; Chiu, 1997) can easily find fuzzy clusters to establish the number of fuzzy rules and the rule premises. Type-2 TSK FLS was presented in 1999 (Liang & Mendel, 1999).

## **1.2 Basic terminology**

### **1.2.1 Subtractive Clustering**

Subtractive clustering method is a fast clustering method designed for high dimension problem with moderate number of data points, because its computation grows linearly with the data dimension and as the square of the number of data points. Subtractive clustering operates by finding the optimal data point to define a cluster center based on the density of surrounding data points.

Using the symbols displayed in Table 1.1, where  $r_a, \eta, \underline{\varepsilon}$  and  $\bar{\varepsilon}$  are positive constants. It is possible to control the subtractive clustering method.

Consider a group of data points  $\{x^1, x^2, \dots, x^w\}$  for a specific class. The M dimensional feature space is normalized so that all data are bounded by a unit hypercube. Calculate potential  $P_i$  for each point as follow:

$$P_i = \sum_{j=1}^w e^{-\alpha \|x^i - x^j\|^2} \quad (1.1)$$

with  $\alpha = \frac{4}{r_a^2}$ .

Thus, the measure of potential for a data point is a function of its distance to all other data points. A data point with many neighboring data points will have a high potential value. After the potential of every data computed, cluster counter  $k=1$ , the data point  $x^i$  with the maximum potential  $P_k^* = P_1^*$  is selected as the first cluster center  $x_k^* = x_1^*$ . Then revise the potential of each data point  $x^i$  by the formula

$$P_i = P_i - P_k^* e^{-\beta \|x^i - x^k\|^2} \quad (1.2)$$

with  $\beta = \frac{4}{r_b^2}$  and  $r_b = \eta * r_a$

Pick up data point  $x^t$  with the current maximum potential  $P_t$  as the candidate for the next cluster center. The process of acquiring new cluster center and revising potentials use the following criteria:

if  $P_t > \bar{\varepsilon} P_1^*$

Accept  $x^t$  as the next cluster center, cluster counter  $k = k + 1$ , and continue.

else if  $P_t < \underline{\varepsilon} P_1^*$

Reject  $x^t$  and end the clustering process.

Table 1.1 Symbols for the subtractive clustering method

Symbol	Definition
$x^i, x^l$	$i$ th and $l$ th normalized data vector of both input and output dimensions in an $n$ dimensional feature space, $i \in [1, w]$ , $l \in [1, w]$
$x_i^j$	$j$ th element in the vector $x^i$ , $x_i^j = \frac{x_j^i - \min(x_j^i)}{\max(x_j^i) - \min(x_j^i)}$ , $j \in [1, n]$
$w$	Number of data points
$P_i$	Potential value of data vector $x^i$
$r_a$	Hypersphere <i>cluster radius</i> in data space -- defines a neighborhood, data points outside this radius has little influence on the potential
$r_b$	Hypersphere <i>penalty radius</i> in data space -- defines the neighborhood which will have the measurable reductions in potential
$\eta$	Squash factor $\eta = \frac{r_b}{r_a}$
$\varepsilon_+$	Reject radio -- specifies a threshold for the potential above which we will definitely accept the data point as a cluster center
$\varepsilon_-$	Accept radio -- specifies a threshold below which we will definitely reject the data point

else

Let  $d_{\min}$  =shortest of the distances between  $x^t$  and all previously found cluster centers.

$$\text{if } \frac{d_{\min}}{r_a} + \frac{P_t}{P_1^*} \geq 1$$

Accept  $x^t$  as the next cluster center. Cluster counter  $k = k + 1$ , and continue.

else

Reject  $x^t$  and set  $P_t = 0$ .

Select the data point with the next highest potential as the new candidate cluster center and retest.

### 1.2.2 Type-1 TSK Fuzzy Logic System

The generalized  $k$ th rule of  $m$ -order type-1 TSK Multi-Input Single-Output (MISO) model can be described by the following expression (Demirli & Muthukumaran, 2000):

IF  $x_1$  is  $Q_1^k$  and  $x_2$  is  $Q_2^k$  and ... and  $x_n$  is  $Q_n^k$ ,

$$\text{THEN } Z \text{ is } w^k = \sum_{j=0}^m \sum_{i=0}^m \cdots \sum_{q=0}^m p_{ji\dots q}^k x_1^j x_2^i \cdots x_n^q$$

where  $p_{ji}^k$  represents  $m$ -order coefficients for  $j = 1, 2, \dots, m, i = 1, 2, \dots, m, \dots, q = 1, 2, \dots, m$  in the  $k$ th rule where  $j, i, \dots, q$  are the order of variables.

When  $m = 1$ ,  $Z$  become a first order type-1 model. A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of a system. For a type-1 TSK model, its  $k$ th rule can be expressed as:

IF  $x_1$  is  $Q_1^k$  and  $x_2$  is  $Q_2^k$  and ... and  $x_n$  is  $Q_n^k$ ,

$$\text{THEN } Z \text{ is } w^k = f^k(x_1, x_2, \dots, x_n) = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n$$

where  $x_1, x_2, \dots, x_n$  and  $Z$  are linguistic variables;  $Q_1^k, Q_2^k, \dots$ , and  $Q_n^k$  are type-1 fuzzy sets on universe of discourses  $X_1, X_2, \dots, X_n$ ;  $p_0^k, p_1^k, p_2^k, \dots, p_n^k$  are constant regression parameters.

Because Gaussian basis functions (GBFs) have the best approximation property (Wang and Mendel, 1992), Gaussian functions are usually chosen as the MFs. A type-1 Gaussian MF can be expressed by using formula for the  $v$ th variable:

$$Q_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*}}{\sigma} \right)^2 \right] \quad (1.3)$$



where  $x_v^{k*}$  is the mean of the  $v$ th input feature in the  $k$ th rule for  $v \in [0, n]$ ,  $\sigma$  is the standard deviation of Gaussian MF.

When certain input values  $x_1^0, x_2^0, \dots, x_n^0$  are given to  $x_1, x_2, \dots, x_n$ , the conclusion from a TSK rule  $r_k$  is a crisp value  $w_k^*$ :

$$w_k^* = f^k(x_1^0, x_2^0, \dots, x_n^0) \quad (1.4)$$

having some rule firing strength (weight) defined as

$$\alpha_k = \mu_1^k(x_1^0) \cap \mu_2^k(x_2^0) \cap \dots \cap \mu_n^k(x_n^0). \quad (1.5)$$

$\alpha_k$  is the activation value of weight for the antecedent of the rule  $r_k$ . Moreover,  $\mu_1^k(x_1^0), \mu_2^k(x_2^0), \dots, \mu_n^k(x_n^0)$  are membership grade for fuzzy sets  $Q_1^k, Q_2^k, \dots$ , and  $Q_n^k$  in the rule  $r_k$ . The symbol  $\cap$  is a conjunction operator, which is a T-norm. In this thesis, the conjunction operator is the minimum operator  $\wedge$  or the product operator  $*$ .

The output of a TSK fuzzy system with  $m$  rules can be expressed (using *weighted average aggregation*) as

$$w^* = \frac{\sum_{k=1}^m \alpha_k w_k^*}{\sum_{k=1}^m \alpha_k} \quad (1.6)$$

### 1.2.3 Type-2 TSK Fuzzy Logic System

#### 1.2.3.1 Type-2 Fuzzy Logic

A *type-2 fuzzy set* (Karnik, Mendel & Liang, 1999) is characterized by a fuzzy membership function, *i.e.* the membership value (or membership grade) for each element of this set is a fuzzy set in  $[0, 1]$ , unlike a type-1 fuzzy set where the membership grade is a crisp number in  $[0, 1]$ .

An example of a type-2 *principal MF* is the Gaussian MF depicted in Figure 1.1, whose vertices have been assumed to vary over some interval of value. The *footprint of uncertainty* (FOU) associated with this type-2 MF is a bounded shaded region, in Fig. 1.1.

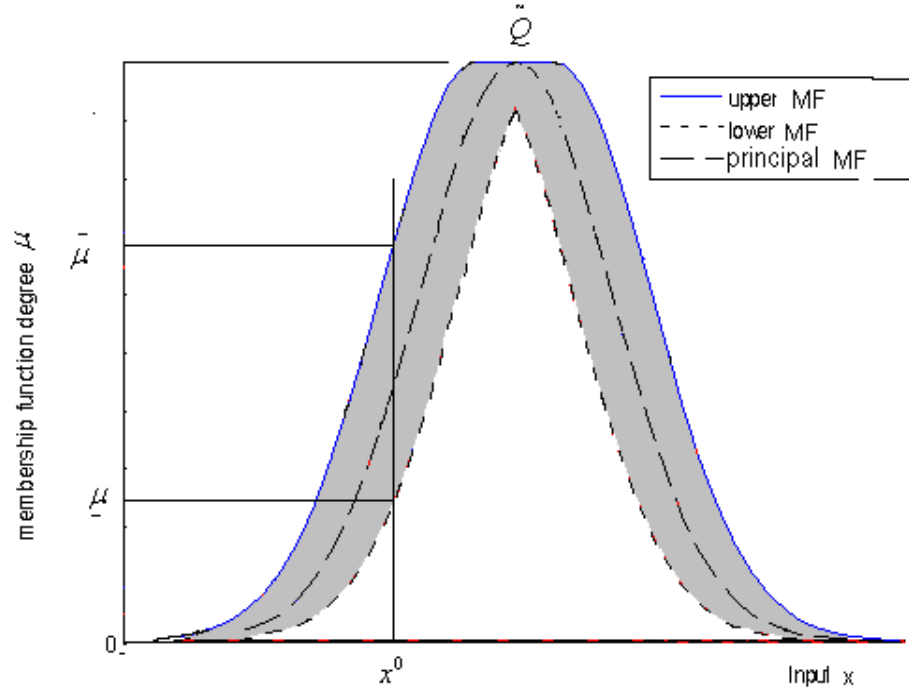


Fig. 1.1 Type-2 Gaussian membership

FOU represents the entire shaded interval type-2 fuzzy set  $\tilde{Q}$ . *Upper MF* and *Lower MF* are two type-1 MFs that are bounds for the FOU of a type-2 set  $\tilde{Q}$ . The intersection of the crisp input  $x^0$  with the *lower MF* is the degree  $\mu$  and with the *upper MF* is the degree  $\bar{\mu}$ . Detailed type-2 fuzzy sets and interval type-2 FLS background material can be found in (Mendel, Hagaras & John, 2006).

### 1.2.3.2 Type-2 TSK Fuzzy Logic System

A generalized  $k$ th rule in the first-order type-2 TSK fuzzy MISO system with a rule base of  $m$  rules, each having  $n$  antecedents, the rule  $r_k$  can be expressed as

$$\begin{aligned} \text{IF } x_1 \text{ is } \tilde{Q}_1^k \text{ and } x_2 \text{ is } \tilde{Q}_2^k \text{ and } \dots \text{ and } x_n \text{ is } \tilde{Q}_n^k, \\ \text{THEN } Z \text{ is } w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n \end{aligned}$$

where  $\tilde{Q}_1^k, \tilde{Q}_2^k, \dots, \tilde{Q}_n^k$  are fuzzy sets on universe of discourses  $X_1, X_2, \dots, X_n$ .  $k \in [1, m]$ , where  $m$  is the total number of rules.  $\tilde{p}_0^k, \tilde{p}_1^k, \dots, \tilde{p}_n^k$  are consequent type-1 fuzzy sets, while  $w^k$  is the consequent of the  $k$ th IF-THEN rule.

A type-2 Gaussian MF can express by using formula for the  $v$ th variable:

$$\tilde{Q}_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*} (1 \pm a_v^k)}{\sigma^k} \right)^2 \right] \quad (1.7)$$

where  $a_v^k$  is spread percentage of mean  $x_v^{k*}$  as shown in Fig. 1.2  $\sigma^k$  is the standard deviation of Gaussian MF of the  $k$ th rule.

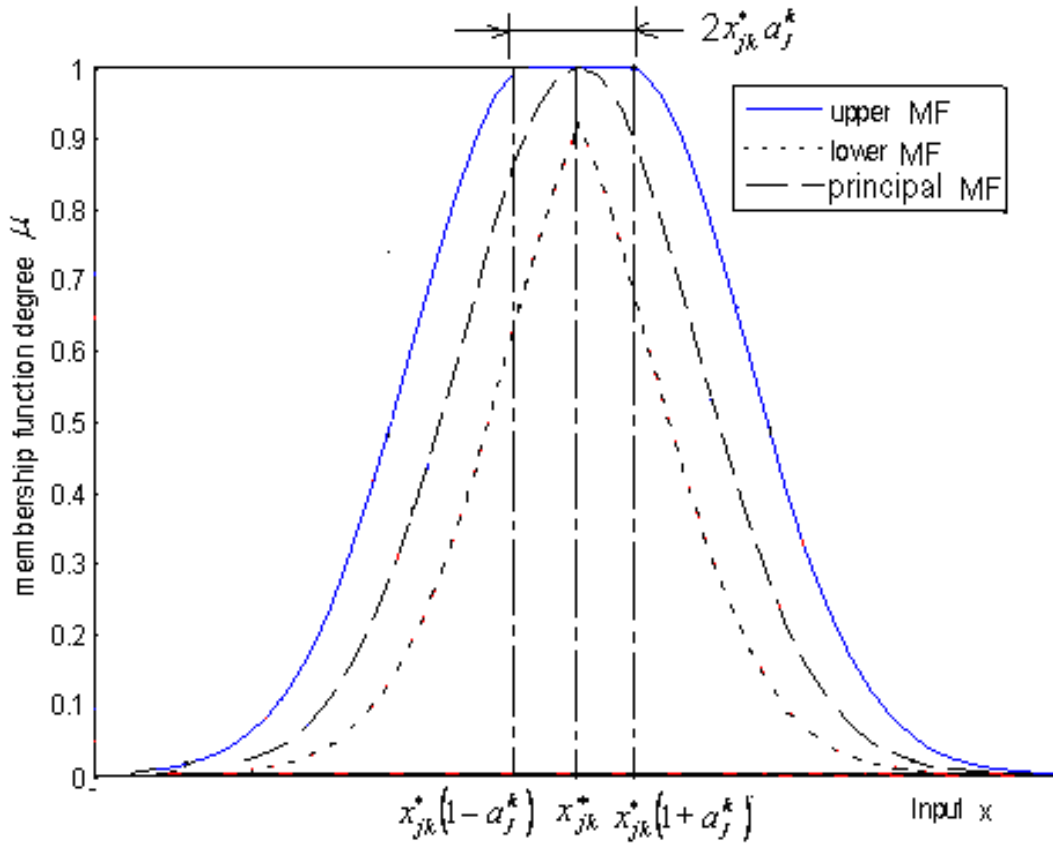


Fig. 1.2 Spread of mean of the  $v$ th variable in the  $k$ th

### 1.2.3.3 Inference Engine

For the most general model of interval type-2 TSK FLS, antecedents are type-2 fuzzy sets and consequents are type-1 fuzzy sets, then consequent parameter  $\tilde{p}_0, \tilde{p}_1, \dots, \tilde{p}_n$  are assumed as convex and normal type-1 fuzzy number subsets of the real line, so that they are fuzzy number., *i.e.*,

$$\tilde{p}_j = \left[ c_j - s_j, c_j + s_j \right] \quad (1.8)$$

where  $c_j$  denotes the centre (mean) of  $\tilde{p}_j$ , and  $s_j$  denotes the spread of fuzzy number  $\tilde{p}_j$ .  $j \in [0, n]$ , where  $j$  is the total number of rules.

MF degree  $\tilde{\mu}_1^k, \tilde{\mu}_2^k, \dots, \tilde{\mu}_n^k$  are interval set,

$$\begin{aligned} \tilde{\mu}_1^k &= \left[ \mu_1^k, \mu_1^k \right] & k = 1, \dots, m \\ \tilde{\mu}_2^k &= \left[ \mu_2^k, \mu_2^k \right] & k = 1, \dots, m \\ & \vdots \\ \tilde{\mu}_n^k &= \left[ \mu_n^k, \mu_n^k \right] & k = 1, \dots, m \end{aligned} \quad (1.9)$$

The *firing set* alters the consequent set for a fired rule in a singleton type-2 FLS. It conveys the uncertainties of the antecedents to the consequent set. The total firing set  $f^k$  for the rule  $r_k$  is interval type-1 set, *i.e.*  $f^k = [f^k, \bar{f}^k]$ . The explicit dependence of  $f^k$  can be computed as:

$$f^k = \mu_1^k(x_1) \cap \mu_2^k(x_2) \cap \dots \cap \mu_n^k(x_n) \quad (1.10)$$

$$\bar{f}^k = \bar{\mu}_1^k(x_1) \cap \bar{\mu}_2^k(x_2) \cap \dots \cap \bar{\mu}_n^k(x_n) \quad (1.11)$$

The consequent  $w^k$  of the rule  $r_k$  is a type-1 fuzzy set because it is a linear combination of type-1 fuzzy sets. It is also an interval set, *i.e.*,

$$w^k = [w_l^k, w_r^k] \quad (1.12)$$

where  $w_l^k$  and  $w_r^k$  are its two end-points, while  $w_l^k$  is the consequent of a type-1 TSK FLS, whose antecedent MF are the lower MFs of the type-2 TSK FLS. Moreover,  $w_r^k$  is the consequent of a type-1 TSK FLS whose antecedent MF are the upper MFs of the type-2 TSK FLS and  $w_l^k$  and  $w_r^k$  can be computed as

$$w_l^k = \sum_{i=1}^P c_i^k x_i + c_0^k - \sum_{i=1}^P s_i^k |x_i| - s_0^k \quad (1.13)$$

$$w_r^k = \sum_{i=1}^P c_i^k x_i + c_0^k + \sum_{i=1}^P s_i^k |x_i| + s_0^k \quad (1.14)$$

$\tilde{w}$  is the *extended output* of a type-2 TSK FLS. It reveals the uncertainty of the output of a type-2 TSK FLS due to antecedent or consequent parameter uncertainties. The interval set of total output  $\tilde{w}$  for  $m$  rules of the system is obtained by applying the Extension Principal (zadeh, 1965; Zadeh, 1975). When interval type-2 fuzzy sets are used for the antecedents, and interval type-1 fuzzy sets are used for the consequent sets of a type-2 TSK rules (Mendel & John, 2001; Mendel & Jahn, 2002; John, 2002),  $\tilde{w}$  can be obtained by

$$\begin{aligned} \tilde{w} &= [w_l, w_r] \\ &= \int_{w^1 \in [w_l^1, w_r^1]} \cdots \int_{w^n \in [w_l^n, w_r^n]} \int_{f^1 \in \left[ \begin{smallmatrix} f^1 & \bar{f}^1 \\ - & \end{smallmatrix} \right]} \cdots \int_{f^n \in \left[ \begin{smallmatrix} f^n & \bar{f}^n \\ - & \end{smallmatrix} \right]} \frac{1}{\frac{\sum_{k=1}^m f^k w^k}{\sum_{k=1}^m f^k}} \end{aligned} \quad (1.15)$$

Hence  $\tilde{w} = [w_l, w_r]$  is an interval type-1 set. To compute  $\tilde{w}$ , its two end-points  $w_l$  and  $w_r$  must be computed. Let the value of  $f^k$  and  $w^k$  that are associated with  $w_l$  be denoted  $f_l^k$  and  $w_l^k$ , respectively, and those associated with  $w_r$  be denoted  $f_r^k$  and  $w_r^k$ . The two endpoints  $w_l$  and  $w_r$  can be obtained as follows:

$$w_l = \frac{\sum_{k=1}^m f_l^k w_l^k}{\sum_{k=1}^m f_l^k} \quad (1.16)$$

$$w_r = \frac{\sum_{k=1}^m f_r^k w_r^k}{\sum_{k=1}^m f_r^k} \quad (1.17)$$

To compute  $w_l$  and  $w_r$ ,  $f_l^k$  and  $f_r^k$  have to be determined. Moreover  $w_l$  and  $w_r$  can be obtained by using the iterative procedure (Karnik & Mendel, 1998) briefly provided:

Without loss of generality, assume that the pre-computed  $w_r^k$  are arranged in ascending order;  $w_r^1 \leq w_r^2 \leq \dots \leq w_r^m$ , then,

Step 1: Compute  $w_r$  in (1.14) by initially setting  $f_r^k = (f_r^k + \bar{f}^k)/2$  for  $k = 1, \dots, m$ , where  $f_r^k$

and  $\bar{f}^k$  have been previously computed using (1.10, 1.11), respectively, and let  $w_r' \equiv w_r$ .

Step 2: Find  $R$  ( $1 \leq R \leq m-1$ ) such that  $w_r^R \leq w_r' \leq w_r^{R+1}$ .

Step 3: Compute  $w_r$  in (1.17) with  $f_r^k = f_r^k$  for  $k \leq R$  and  $f_r^k = \bar{f}^k$  for  $k > R$ , and let

$$w_r'' \equiv w_r.$$

Step 4: If  $w_r'' \neq w_r'$ , then go to Step 5. If  $w_r'' = w_r'$ , then stop and set  $w_r'' \equiv w_r$ .

Step 5: Set  $w_r' = w_r''$ , and return to Step 2.

The procedure for computing  $w_l$  is very similar to the one just given for  $w_r$ . Replace  $w_r^k$  by  $w_l^k$ , and, in Step 2 find  $L$  ( $1 \leq L \leq m-1$ ) such that  $w_l^L \leq w_l' \leq w_l^{L+1}$ . Additionally, in Step 3, compute  $w_l$  in (1.16) with  $f_l^k = f_l^k$  for  $k \leq L$  and  $f_l^k = \bar{f}^k$  for  $k > L$ .

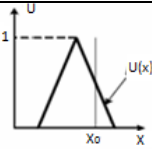
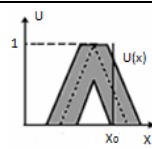
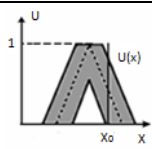
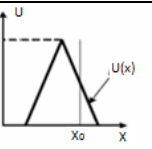
In an interval type-2 TSK FLS, output  $\tilde{w}$  is an interval type-1 fuzzy set, so the crisp output  $w^*$  of any interval type-2 TSK FLS can be obtained by using the average value of  $w_l$  and  $w_r$ . Hence, the crisp output of type-2 TSK FLS is

$$w^* = \frac{w_l + w_r}{2} \quad (1.18)$$

### 1.3 Comparison between type-1 and type-2 TSK Fuzzy Logic System

Type-1 and type-2 TSK FLSs are characterized by IF-THEN rules and no defuzzification is needed in the inference engine, but they have different antecedent and consequent structures. Assuming FLSs with  $m$  rules and  $n$  antecedents in each rule, a type-1 TSK FLS is compared with a type-2 TSK FLS in Table 1.2.

Table 1.2 Comparison between type-1 and type-2 TSK

TSK FLS		Type-1	Type-2		
			Model 1	Model 2	Model 3
Structure	Antecedents	Type-1 fuzzy set	Type-2 fuzzy set	Type-2 fuzzy set	Type-1 fuzzy set
					
	Consequent parameters	Crisp number	Fuzzy number	Crisp number	Fuzzy number
Output		A crisp point	An interval set of output		
			A point output		
Number of design parameters		$(3p+1)M^*$	$(5p+2)M^*$	$(4p+1)M^*$	$(4p+2)M^*$

\* There are  $M$  rules and each rule has  $p$  antecedents in the fuzzy system

Apparently, a type-2 TSK FLS has more design degrees of freedom than does a type-1 TSK FLS because its type-2 fuzzy sets are described by more parameters than type-1 fuzzy sets (Mendel, 2001). This suggests that a type-2 TSK FLS has the potential to outperform a type-1 TSK FLS because of its larger number of design degrees of freedom. Type-2 system is able to model more complex input-output relationship to achieve the universal approximation property.

Type-2 FLS has great capability to model uncertainties in the data sets and minimize their effects (Ren, Balazinski & Baron, 2011).

## 1.4 Preliminary Research

Subtractive clustering based type-2 TSK fuzzy system identification algorithm (as shown in Fig. 1.3) was proposed (Ren, 2006; Ren, Baron & Balazinski, 2006) to directly extend a type-1 TSK FLS to its type-2 counterpart with emphasis on interval set. The Type-1 Gaussian MFs is used as principal MFs to expand type-1 TSK model to type-2 TSK model. The proposed type-2 TSK modeling identification algorithm has the following steps:

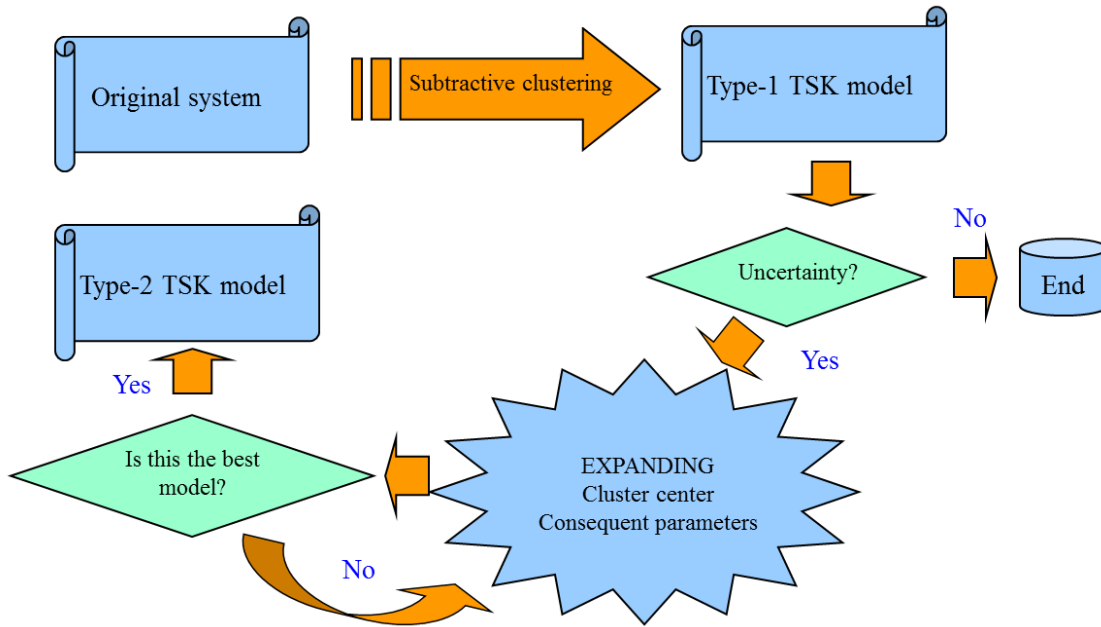


Fig. 1.3 Subtractive clustering based type-2 TSK fuzzy system identification

- Step 1: Use Chiu's subtractive clustering method combined with least squares estimation algorithm to pre-identify a type-1 fuzzy model from input/output data.
- Step 2: Calculate the root-mean-square-error (RMSE) of the type-1 fuzzy model. If RMSE is bigger than expected error limitation, go to Step3. If not, end program, which means that the type-1 model is acceptable, and it is not needed to use type-2 TSK model.



Step 3: Use type-1 Gaussian MFs as principal MFs to expand type-1 TSK model to type-2 TSK model:

- Spread cluster center to expand premise MFs from type-1 fuzzy sets to type-2 fuzzy sets as depicted in Figure 1.2. The cluster center is fuzzy number where

$$\tilde{x}_v^{k*} = [x_v^{k*}(1 - a_v^k), x_v^{k*}(1 + a_v^k)] \quad (1.19)$$

- The deviation for each rule varies from each other in the fuzzy model to get the best model where constant  $\sigma^k$  is replaced by  $\sigma_v^k$  as shown in Fig. 1.4.

$$\tilde{Q}_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*}(1 \pm a_v^k)}{\sigma_v^k} \right)^2 \right] \quad (1.20)$$

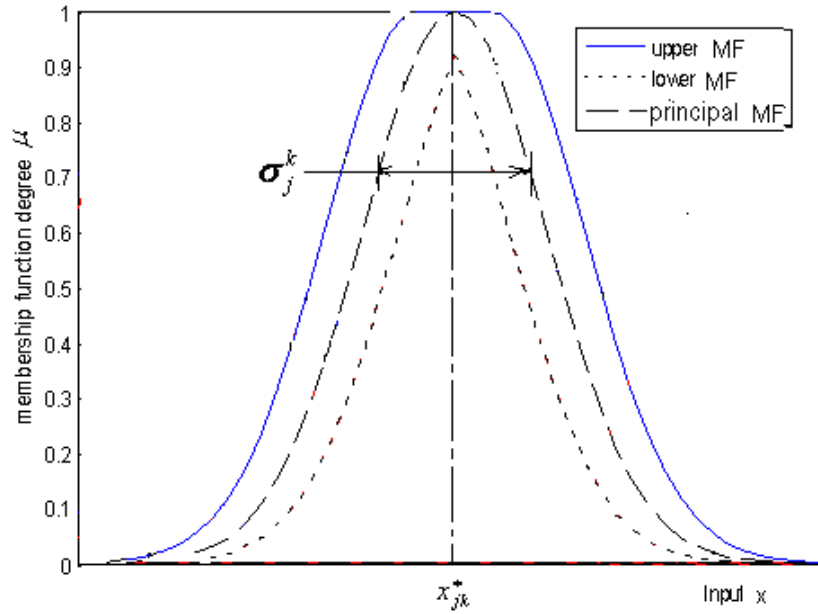


Fig. 1.4 Standard deviation of Gaussian MF

- Spread the parameters of consequence to expand consequent parameters from constants to fuzzy numbers:

$$\tilde{p}_j^k = p_j^k (1 \pm b_j^k) \quad (1.21)$$

where  $b_j^k$  is the spread percentage of fuzzy number  $\tilde{p}_j^k$ .

Step 4: By using type-2 fuzzy inference engine, compute the interval value of the consequent for each variable and obtaining the two end-points of interval set and average value of output.

Step 5: Calculate RMSE of this type-2 model. Choose the model with least RMSE.

## **CHAPTER 2      LITERATURE REVIEW**

Because of their universal approximation property, TSK FLSs described in CHAPTER 1 are widely developed and used. This section reviews the studies about type-1 and type-2 TSK FLSs. Uncertainties in machining are summarized to point out the need of an advanced artificial intelligent methodology – type-2 fuzzy logic modeling which handle uncertainty effectively and minimize the effect on products.

### **2.1 Type-1 TSK Fuzzy Logic System**

The proposed linguistic approach by Zadeh (Zadeh, 1968; Zadeh, 1973), following the first paper “Fuzzy Sets” in 1965, has the capability to model complex system behavior in such a qualitative way that the model is effective and versatile in capturing the behavior of ill-defined systems with realistic approximations. Motivated by these ideas of “fuzzy algorithm” and “linguistic analysis”, Mamdani first applied fuzzy logic (FL) to control engineering (Mamdani & Assilian, 1974). This topic is known as fuzzy algorithmic control or linguistic control. The main problem with fuzzy control is the design of a fuzzy controller where we usually take an expert-system-like approach. That is, we derive fuzzy control rules from the human operator’s experience and/or engineer’s knowledge, which are mostly based on their qualitative knowledge of an objective system. A set of fuzzy control rules is a linguistic model of human control actions which is not based on a mathematical description of human control actions but is directly based on a human way of thinking about plant operation.

Zadeh’s proposal of linguistic approach is effective and versatile in modeling ill-defined systems with fuzziness or fully defined systems with realistic approximations. Later it is expanded into fuzzy systems modeling as a qualitative modeling (Tong, 1979; Pedrycz, 1984; Takagi & Sugeno, 1985; Takagi & Sugeno, 1986; Trojan, Kiszka, Gupta & Nikiforuk, 1987; Sugeno & Kang, 1988; Sugeno & Tanaka, 1991; Sugeno & Yasukawa, 1993).

TSK FLS has attracted many attentions. The main difference with more traditional fuzzy rules (Mamdani & Assilian, 1974) is that the consequent of the rules are a function of the input variables values.

Table 2.1 Comparisons of Mamdani FLS and TSK FLS

		Mamdani FLS	TSK FLS
Similarity		<ul style="list-style-type: none"> <li>Both are characterized by IF-THEN rules</li> <li>Both have same antecedent structures, MFs are fuzzy sets</li> <li>Both are universal approximator</li> </ul>	
Difference	structures of consequent	a fuzzy set	a function
	inference	When all of the fuzzy sets are type-1, then its output is a type-1 set that is then defuzzified to obtain a type-0 set.	When all of the fuzzy sets are type-1, then its output is a type-0 set. No defuzzification is needed
	applicability	Uncertainty can be accounted for both antecedent and consequent membership functions	Uncertainty can be accounted for just in the antecedent membership functions
Advantage and Disadvantage	Advantage	<ul style="list-style-type: none"> <li>Intuitive.</li> <li>Widespread acceptance.</li> <li>Well suited to human input.</li> </ul>	<ul style="list-style-type: none"> <li>Computationally efficient.</li> <li>Works well with linear techniques (e.g., PID control).</li> <li>Works well with optimization and adaptive techniques.</li> <li>Guaranteed continuity of the output surface.</li> <li>Well-suited to mathematical analysis.</li> <li>More number of design parameters for each rule. It is possible to need fewer rules</li> </ul>
	Disadvantage		More computationally efficient but lose linguistic interpretability.

TSK approach has been demonstrated to have a powerful representative capability, being able to describe non-linear mappings using a small number of simple rules. There is a need to develop a semi-automatic method to obtain those models based on sets of input-output data as shown in Fig. 2.1.

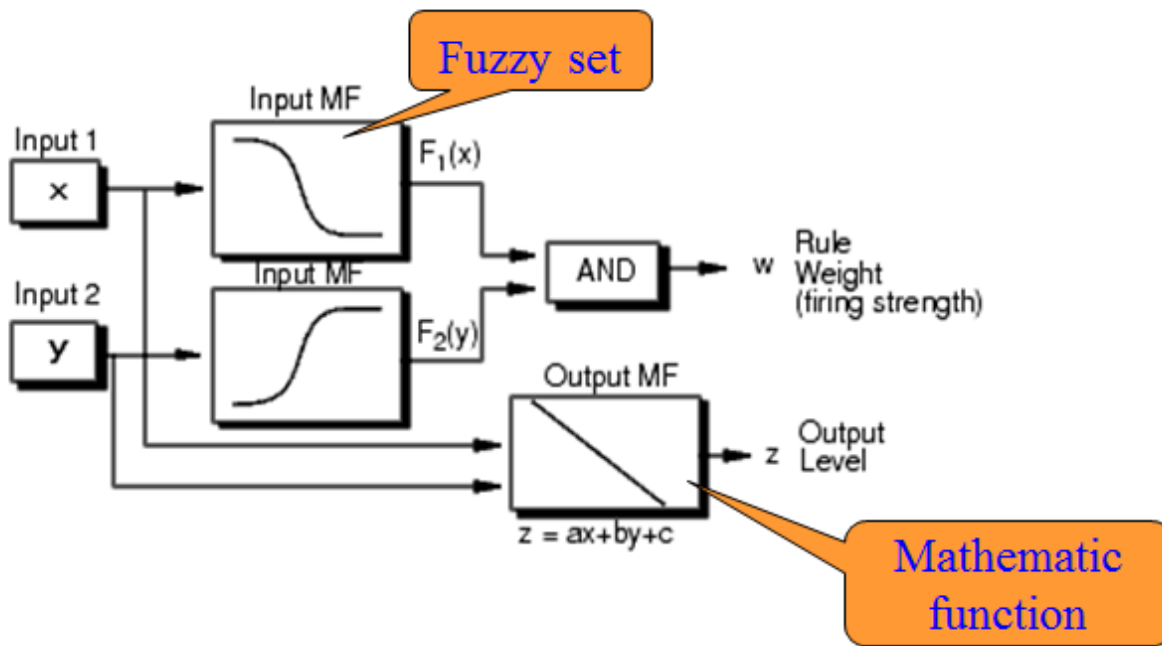


Fig. 2.1 TSK fuzzy logic system

The generality of TSK models makes the *data driven identification* very complex. A fuzzy model consists of multiple rules, each rule containing a premise part and a consequent part. The premise part specifies a certain input subspace by a conjunction of fuzzy clauses that contain the input variables. The consequent part is a linear regression model. The identification task includes two subtasks: *structure identification* and *parameter identification* as shown in Fig. 2.2. The former is the determination of the number of rules and the variables involved in the rule premises, while the latter is the estimation of the membership function parameters and the estimation of the consequent regression coefficients.

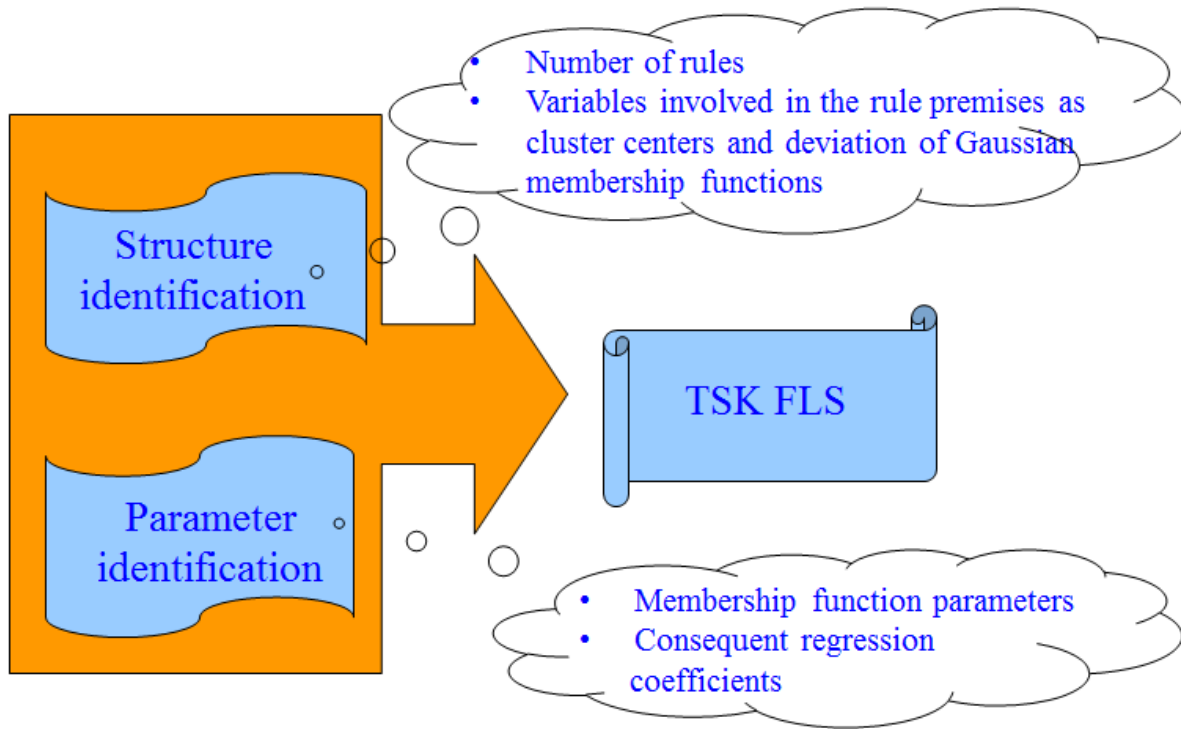


Fig. 2.2 TSK FLS identification

The TSK FLSs is a more compact and computationally efficient representation than a Mamdani FLSs. It lends itself to the use of adaptive techniques for constructing fuzzy models. These adaptive techniques can be used to customize the membership functions so that the fuzzy system best models the data.

Clustering methods are proposed to identify natural grouping of data from a large data set such that a concise representation of system's behavior is produced. Yager & Filev (Yager & Filev, 1992; Yager & Filev, 1994) developed the mountain method for estimating cluster centroids. Mamdani and Assilian (Mamdani & Assilian, 1997), Bezdek (Bezdek, 1974) and Bezdek et al. (Bezdek, Hathaway, Sabin & Tucker, 1987) proposed a variety of clustering algorithms, including hierarchical, k-means, and fuzzy c-means algorithms. The initial selection of cluster details has been made automatic (Emami, Truksen & Goldenberg, 1998). Chiu's approach known as subtractive clustering reduce the computational complexities. In the literature, different modeling techniques can be found as for example (Babuska, 1998).

TSK FLSs are widely used for model-based control and model-based fault diagnosis. This is due to the model's properties of, on one hand being a general nonlinear approximator that can

approximate every continuous mapping, and on the other hand, being a piecewise linear model that is relatively easy to interpret (Johansen & Foss, 1995) and whose linear submodels can be exploited for control and fault detection (Füssel, Ballé & Isermann, 1997).

Zero order and first order TSK FLSs based on clustering method suffer the curse of dimensionality – the number of rules increases exponentially with the number of input variables and the number of MFs per variable (Bellman, 1961). For example, if there are  $m$  MFs for each input variable and a total of  $n$  input variables for the problem, the total number of fuzzy rules is  $m^n$ , as shown in Fig. 2.3.

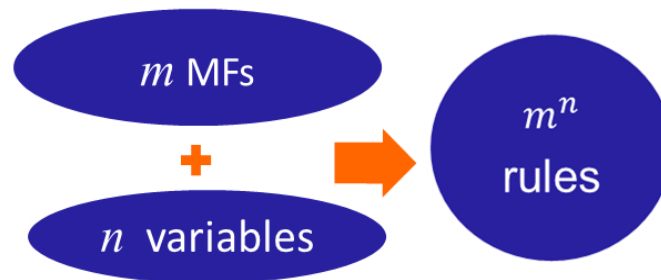


Fig. 2.3 The curse of dimensionality

High order consequent TSK rules can reduce drastically the number of rules needed to perform the approximation, and improve transparency and interpretation in many high dimensional situations. High order TSK fuzzy systems were applied in numerous fields:

- system identification (Demirli & Muthukumaran, 2000);
- machining process modeling (Al-Wedyan, Demirli & Bhat, 2001);
- medical decision support (Song, Ma & Kasabov, 2003);
- temperature control (Galan, 2004);
- function approximation (Kim K. et al, 2004; Herrera, Pomares, Rojas, Valenzuela & Prieto, 2005);
- fuzzy controller (Savkovic-Stevanovic, 2004; Nu & Zhang, 2005, Xie & Jamwal, 2010);

- creating personalized models (Song, Ma & Kasabov, 2005);
- pattern recognition (Gupta, 2005);
- diagnosis of tool wear (Jantunen, 2006);
- noise cancellation (Theochaires, 2006);
- drilling performances (Nandi & Davim, 2009);
- modeling and control of nonlinear dynamical systems (Tanaka, Yoshida, Ohtake & Wang, 2009)
- regression problem and one system identification task (Tsakonas & Gabrys, 2011) and etc.

## **2.2 Type-2 Fuzzy Logic and Fuzzy Logic Systems**

### **2.2.1 Type-2 Fuzzy Set and Fuzzy Logic**

Type-2 fuzzy sets were also introduced by Zadeh (Zadeh, 1968). The main concept of type-2 fuzzy logic is that “words mean different things to different people”. Thus, there are uncertainties associated words. Based on Extension Principal (Zadeh, 1975), Algebraic structure of type-2 fuzzy sets were studied (Mizumoto & Tanaka, 1976; Roger, 1980; Mizumoto & Tanaka, 1981; Nminen, 1997). Debois & Prade (Debois & Prade, 1978; Debois & Prade, 1979; Debois & Prade 1980) discussed fuzzy valued logic depended on minimum conjunction and gave a formula for the composition of type-2 relations.

J. M. Mendel, R. I. John and their collaborators extended previous studies and obtained practical algorithms for conjunction, disjunction and complementation operations of type-2 fuzzy sets. They also developed a general formula for the extended composition of type-2 relations which is considered as an extension of the type-1 composition (Karnik & Mendel, 1998). Based on this formula, they established a complete type-2 fuzzy logic theory with the handling of uncertainties (Mendel, 2001). The characterization in the definition of type-2 fuzzy sets uses the notion that type-1 fuzzy sets can be thought of as a first order approximation to uncertainty and, therefore, type-2 fuzzy sets provides a second order approximation. They play an important role in modeling uncertainties that exist in fuzzy logic systems (John, 1998), and are becoming increasingly important in the goal of “Computing with Words” (Zadeh, 1996) and



“Computational Theory of Perceptions” (Zadeh, 1999). John’s embedded interval valued type-2 fuzzy sets was introduced and it made type-2 fuzzy sets easy to understand and explain (Mendel & John, 2002; John & Coupland, 2007). Because of its larger number of design parameters for each rule, it was believed that type-2 FLS have the potential to be used in control (Hagras, 2007) and other areas where a type-1 model may be unable to perform well (Wu & Tan, 2006).

### 2.2.2 Type-2 TSK Fuzzy Logic systems

Type-2 TSK FLSs are presented in 1999 (Liang & Mendel, 1999), and type-2 TSK FLSs have the potential to be used in control and other areas where a type-1 TSK model may be unable to perform well because of its large numbers of design parameters. In 2001, Mendel published his famous book “*Uncertain Rule-Based Fuzzy Logic Systems – Introduction and New Directions*” (Mendel, 2001) including the architecture, inference engine and design method of first-order interval TSK FLS. First-order interval non-singleton type-2 TSK FLS was in 2006 (Mendez, Castillo & Hernandez, 2006). Generalized interval type-2 TSK FLS and high order interval type-2 TSK FLS are proposed in 2008 (Ren, Baron & Balazinski, 2008)

Dereli et al. reviewed industrial applications of type-2 fuzzy sets and systems (Dereli, Baykasoğlu, Altun, Durmusoğlu & Turksen, 2011). Here, recent researches on type-2 TSK fuzzy systems are categorized as follows:

- type-2 fuzzy modeling (Liang & Mendel, 1999; Ren, Baron, Jemielniak & Balazinski, 2010; Fadali, Jafarzadeh & Nafeh, 2010; Mendez & Hernandez, 2010, Ren, Balazinski & Baron, 2011; Ren, Balazinski & Baron, 2012);
- decision feedback equalizer for nonlinear time-varying channels (Liang & Mendel, 2000);
- function approximation (Ren, 2006; Ren, Baron & Balazinski, 2006);
- prediction of the transfer bar surface temperature at finishing scale breaker entry zone (Mendez, Castillo & Hernandez, 2006);
- system identification (Ren, Qin, Baron, Birglen & Balazinski, 2007; Garcia, 2009; Abiyev, Kaynak, Alshanableh & Mamedov, 2011);
- noise filtering (Yildirim, Basturk & Yuksel, 2008; Ren, Baron & Balazinski, 2009 (b));

- tool wear condition monitoring (Ren , Baron & Balazinski, 2009 (a));
- uncertainty estimation on cutting force (Ren , Baron & Balazinski, 2009 (c));
- fuzzy controller (Biglarbegian, Melek & Mendel, 2010; Kayacan, Oniz, Aras, Kaynak & Abiyev, 2011, Chen, 2011; Biglarbegian, Melek & Mendel, 2011);
- acoustic emission signal feature analysis (Ren, Baron, Balazinski & Jemielniak, 2010(a));
- short-term power load forecasting (Yao, Jiang & Xiao, 2011) and etc.

## 2.3 Uncertainty in machining

Machining is the most important process in the manufacturing of almost all metal products. Variation reduction and quality improvement is a very important and challenging topic in the modern machining. Sources of uncertainties in machining are both in physical machining system and machining process.

### 2.3.1 Machining systems

Figure 2.4 illustrates a physical machining system, in which the material is being removed by the tool during the cutting process (Zhang, Hwang & Ratnakar, 1993).

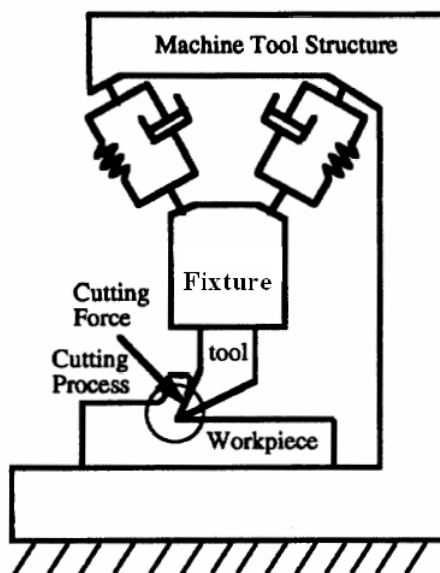


Fig. 2.4 Representation of a physical machining system

The structural dynamics of the machine tool is represented by a two-degree-of-freedom model. As illustrated, the cutting area, or the nominal chip load, serves as the input of the machining system. The cutting force is generated when the material is removed to form chips. The generated cutting force excites the machine tool structure. The dynamic variation of tool displacement changes the chip load through a feedback loop.

### **2.3.2 Sources of uncertainties in machining systems**

Given a particular machine tool, estimating beforehand the errors in features for the parts produced by that machine is not a clearly defined process. Although there are general guides for reporting uncertainties in experiments (International Organization for Standardization, 1986; Taylor & Kuyatt, 1994; American National Standards Institute, 1997), there have been no published practical case studies on how to estimate uncertainties of errors of machined part features in production environments. Various studies have discussed different aspects of the problem of uncertainties in machining system.

- Uncertainties intrinsic to the machine, considered as kinematics error (Ogata K., 1970), structural inaccuracies (Shin & Wei, 1992; Wilhelm, Srinivasan & Farabaugh, 1997; Chatterjee, 1997), etc.
- Uncertainties due to environmental effects which relate to the influence of the atmospheric conditions on the machine tool accuracy and repeatability (Bryan, 1990; Hatamura Y. et al. 1993).
- Uncertainties due to measurement, even the unit of measurement (Swyt, 2001).
- Uncertainties due to the installation. Datum establishment uncertainty is one of the major sources of geometric errors in machining (Wu, 1998).
- Uncertainties due to timing error and computational errors (Ramesh, Mannan & Poo, 2002; Castroa & Burdekin, 2005)

### **2.3.3 Sources of uncertainties in machining processes**

A machining process is typically a discrete and multi-operational process with multivariate quality characteristics. The increasing interest in variation reduction and quality improvement has

heightened the need for studying uncertainties in machining processes.

Different cutting strategies yield different levels of variation and a lot of variation can occur even with the same configuration. Reasons for machining being difficult to analyze and characterize can be summarized as follows.

- The strain rate is extremely high compared to that of other fabrication processes.
- The process varies considerably depending on the part material, temperature, cutting fluids, etc. The quantitative evaluation of chip formation is more difficult than for the pure deformation model because of its greater complexity (Baker, 2005).
- The process varies considerably depending on the tool material, temperature, chatter and vibration (Jetly, 1984), etc.
- The primary factor affecting machining accuracy is the uncertainty of the thickness of cut - the undeformed thickness of the work material removed at the top portion of the round (Ikawa Shimada & Tanaka, 1992).

Modeling activities for manufacturing processes generally include identifying the relevant inputs between these inputs to produce the desired outputs using numerical and/ or analytical algorithms. Whether considering a measurement or model prediction, however, the result is incomplete if it is not accompanied by a quantitative assessment of its uncertainty (Schmitz, Jaydeep, Kim & Abbas, 2011). For machining processes, there is a lack of system level models and mathematical tools to understand variation propagation so as to support fully the integration of design, manufacturing and quality improvement. As a result, a large number of iterative design changes and human interventions are required due to a poor system response to uncertainties.

## 2.4 Summary

The foregoing literature survey demonstrates the progress made in both type-1 TSK FLS and type-2 FL & TSK FLS to date, as well as the sources of uncertainty in machining. The review showed that:

- type-1 TSK FLS is at much more studied than the type-2. Type-2 TSK FLS is a new generation of TSK FLS, which means that fuzzy experts have to learn about type-1 FL before type-2 FL, and develop type-1 TSK FLS as far as possible. Then they see the shortcomings of type-1 TSK FLS, and apply type-2 TSK FLS to situations where uncertainties abound;
- nowadays, type-2 fuzzy system is the only artificial intelligent method which can handle

the uncertainty in the modeled system;

- uncertainties in machining influence numerous process outputs. To minimize them, a systematic approach should be used to control the accumulation of variation from all machining operation - machine structure and behavior, machining process setup, adaptive machining, process verification and outcomes monitoring. Advanced methodologies need to be developed for the information processing to monitor the process and to identify the faulty stations and their root caused based on limited measurements;
- because the information obtained during the machining process is vague, incomplete or imprecise, conventional methods need a large number of cutting experiments and additional assumptions in many circumstances for effective uncertainty handling. These requirements reduce the reliability of the models and increase cost.
- artificial intelligence methods have played an important role in modern modeling and monitoring systems;
- applications of type-2 fuzzy logic in mechanical engineering are limited. None of research had been done on application of type-2 fuzzy logic on uncertainty estimation in machining
- high order type-1 TSK FLSs are developed to overcome the problem of dimensionality.
- similar to type-1 rules of type-1 TSK system, first order type-2 TSK rules suffer the curse of dimensionality – the number of rules increases exponentially with the number of input variables and the number of MFs per variable.

## CHAPTER 3      THESIS STRUCTURE

In this chapter, research objectives are presented after defining the problems in previous research. The following chapters of this thesis are composed by three published paper to show the research progress on TSK FLSs (type-1, first order interval type-2 and high order interval type-2) and their application on acoustic emission modeling.

### 3.1 Research motivation and problem statement

In 2006, Subtractive clustering based type-2 TSK fuzzy system identification algorithm was proposed and presented in The 25th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2006). That was the first time that we joined type-2 fuzzy logic community, shared and exchanged our knowledge and experience with the colleagues around the world. We realized that type-2 fuzzy logic had become a very strategic and active research area around the world. It seems that type-2 FL moves in progressive ways where type-1 FL is eventually replaced or supplemented by type-2 FL.

Based on the literature review and our previous studies, the type-2 fuzzy logic should be very suitable to identify the uncertainty in machining which has direct effect on products. And none of research had been done on this subject.

Even we had proposed the type-2 TSK fuzzy system identification algorithm, but it was only used for a function approximation. There are a lot of challenging problems waiting to overcome. Here is a list of the questions which would like to be compiled during this Ph. D. research period.

- There is not any real application for the type-2 TSK fuzzy system identification algorithm based on subtractive clustering. Can it be used to predict uncertainties in mechanical engineering and to solve the real problems?
- There is no guarantee that a type-2 TSK FLS have the potential to outperform its type-1 counterpart and it is difficult to determine the search range for spreading percentage of cluster centers and consequence parameters to obtain an optimal type-2 model. Will the type-2 model be better than its counterpart when we apply our algorithm to the experimental studies?

- Is it possible to develop high order FLS to solve the problem of the curse of dimensionality in a type-2 TSK FLS?

## 3.2 Research Objectives

The main objective of this thesis is to apply the type-2 TSK identification algorithm based on subtractive clustering method to a real problem with uncertainty estimation in machining, and develop a high order type-2 fuzzy system to solve the problem of curse of dimensionality.

The research briefs are:

- to show the differences between type-2 TSK system and its type-1 counterpart
- to demonstrate that type-2 modeling has better performance if it is so the case
- to propose a reliable type-2 fuzzy tool condition estimation method based on information of uncertainty in machining
- to develop a generalized type-2 TSK FLS and high order TSK FLS - architecture, inference engine and design method.
- to demonstrate that high order interval type-2 TSK FLS has the capability to overcome the problem of dimensionality.

## 3.3 Thesis Chapters

CHAPTER 1 introduces the basic knowledge of the research objective of this thesis, while CHAPTER 2 presents the literature review of previous studies. The following chapters of this thesis are organized as:

CHAPTER 4 is the paper “Fuzzy Identification of Cutting Acoustic Emission with Extended Subtractive Cluster Analysis” (Ren, Baron & Balazinski, 2012), published in the journal “Nonlinear Dynamics”, (volume 67, no. 4, pp. 2599-2608, 2012. DOI: 10.1007/s11071-011-0173-5). This Chapter presents type-1 TSK FLS based on extended subtractive clustering combined with least-square estimation method and its application to fuzzy modeling of machining. An experimental study on fuzzy AE identification in high precision hard turning process is given to prove that the method is efficient and feasible.

CHAPTER 5 is the paper “Type-2 Fuzzy Modeling for Acoustic Emission Signal in Precision Manufacturing” (Ren, Baron & Balazinski, 2011), published in the journal “Modeling and Simulation in Engineering” (volume 2011, no. 696947. DOI:10.1155/2011/696947). In this chapter, a first order interval type-2 TSK FLS and its identification algorithm are presented. Experiments show that the development of the AE signal uncertainty trend corresponds to that of the tool wear. Information from the AE uncertainty scheme can be used to make decisions or investigate the tool condition so as to enhance the reliability of tool wear.

CHAPTER 6 is the paper “High Order Interval Type-2 Takagi-Sugeno-Kang Fuzzy Logic System and Its Application in Acoustic Emission Signal Modeling in Turning Process” (Ren, Balazinski & Baron, 2012), published in “International Journal of Advanced Manufacturing Technology” (2012. DOI: 10.1007/s00170-012-3956-z). This chapter presents the architecture and inference engine of generalized interval type-2 TSK FLS, and design method of higher order interval type-2 FLS as well. An experimental acoustic emission signal modeling using a second order interval type-2 TSK FLS in turning process is given to demonstrate the advantage of high order interval type-2 FLS.

The rest of this thesis is a general discussion and conclusions.

### 3.4 Research Strategy

This thesis is composed by three papers published in international journals, they cover the following topics: type-1 TSK FLS, first order interval type-2 FLS and high order Interval FLS, based on one experimental data set which is a time series AE signal voltage for a high precision turning process.

The adopted research strategy is shown in Fig. 3.1 including the theoretical studies and experimental studies.

For the theoretical study, there is a detailed introduction for type-1 TSK FLS in CHAPTER 4 and first order type-2 TSK FLS in CHAPTER 5 including subtractive clustering based type-2 identification algorithm. In CHAPTER 6, the generalized type-2 TSK FLS is proposed. In the generalized TSK FLS, the antecedent or consequent membership functions are type-2 fuzzy sets and the consequent part a first or higher order polynomial function. The



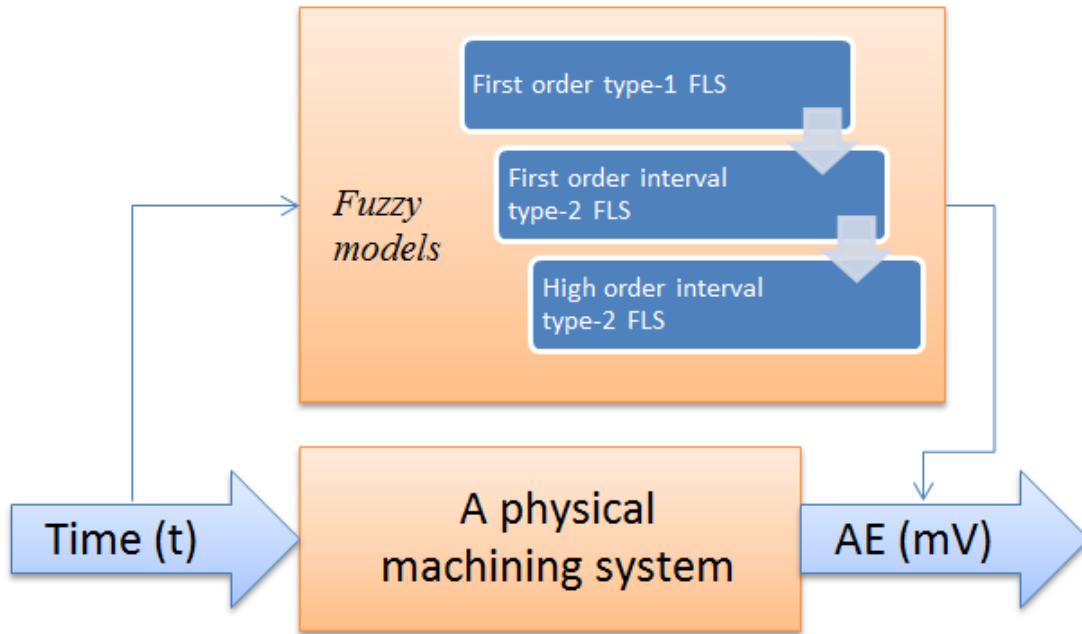


Fig. 3.1 Study strategies

architecture of the generalized type-2 TSK FLS and its inference engine are based on the Mendel's first order type-2 TSK FLS. The design method of high order system is an extension of the subtractive clustering based type-2 TSK FLS identification algorithm.

For the case studies, same data set recorded from the physical machining system is used to identify the type-1, first-order type-2 and high order type-2 fuzzy systems. The aims are listed as follows.

- To Comparing type-2 TSK system and its type-1 counterpart to show the differences between them and demonstrate that type-2 modeling has better performance if it is so the case.
- To predict the uncertainty in AE in precision process and find out if AE uncertainty scheme corresponds to the complex tool wear state development.
- To comparing a second order IT2 TSK FLS with that of first order FLS to show that a second order IT2 TSK FLS can model the AE signal with less rules and similar RMSE.

## CHAPTER 4 FUZZY IDENTIFICATION OF CUTTING ACOUSTIC EMISSION WITH EXTENDED SUBTRACTIVE CLUSTER ANALYSIS

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**Abstract** – This paper presents fuzzy acoustic emission identification in high precision hard turning process based on extended subtractive cluster analysis combined with least-square estimation method. The fuzzy identification method provides a simple way to arrive at a definite conclusion based upon the information obtained with the difficulty in understanding the exact physics of the machining process. The experimental results prove that the proposed method is efficient and feasible.

**Key words** – *fuzzy identification, subtractive clustering, precision machining, acoustic emission, tool wear*

### **Nomenclature:**

AE: acoustic emission

TCM: tool condition monitoring

TSK: Takagi-Sugeno-Kang

FLS: fuzzy logic system

MISO: Multi-input-single -output

MF: membership function

RMS: root-mean-square

## 4.1 Introduction

Acoustic emission (AE) is the class of phenomena whereby transient elastic waves are generated by the rapid release of energy for a localized source or sources within a material, or transient elastic wave(s) so generated (ANSI/ASTM E 610-89). Emissions from process changes, like tool wear, chip formation, can be directly related to the mechanics of the process.

There are different mechanisms that produce AE during cutting process. Fig. 4.1 summarizes application of sensors and sources of AE according to different level of precision and material removal length scale [1]. Along with the scale of precision machining (from conventional to precision, even ultraprecision) becomes finer and closer to the dimensional scale of material properties, microscopic sources become very significant and must be introduced as an important AE sources in precision manufacturing [2].

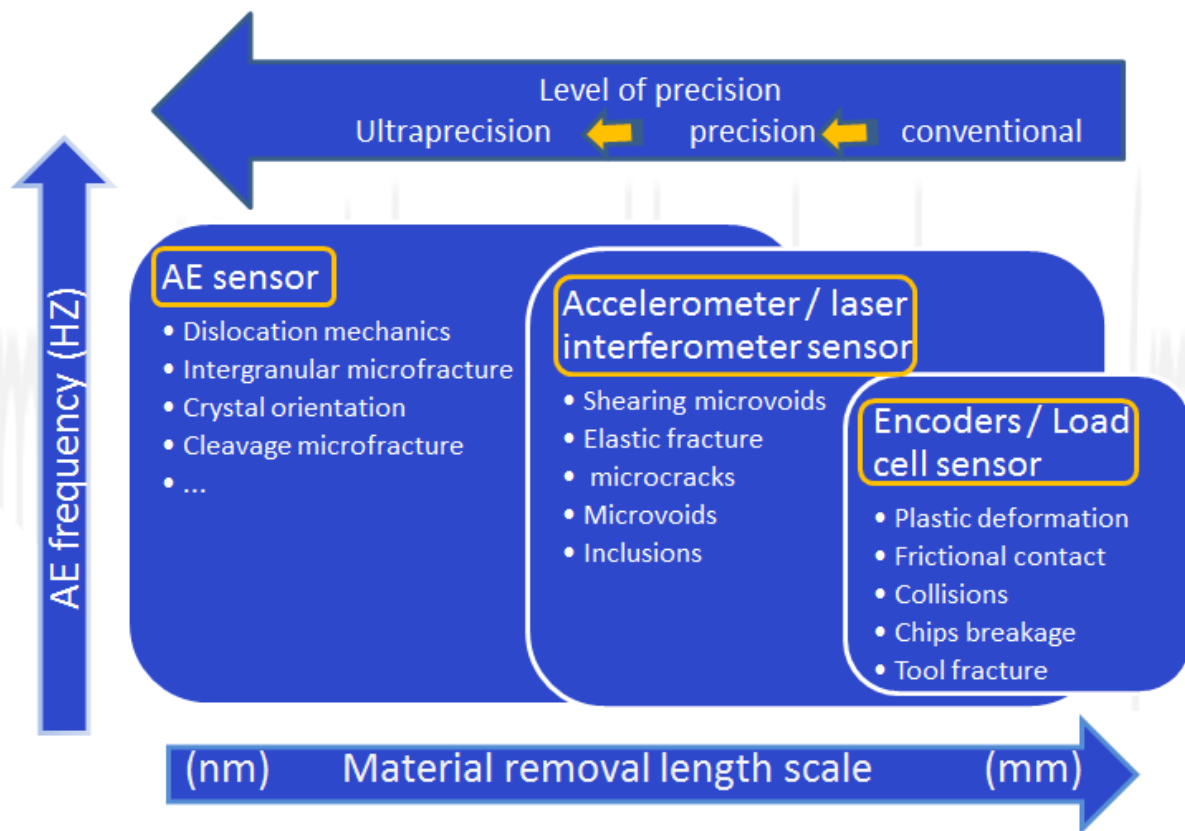


Fig. 4.1 Application of sensors and sources of AE according to different level of precision and material removal length scale

AE based sensing methodologies for tool condition and cutting process monitoring have been studied since 1977 [3]. AE can be effectively used for tool condition monitoring applications because that the emissions from process changes with tool wear, chip formation can be directly related to the mechanics of the process. Also the frequency range of AE is much higher than that of machine vibrations and environmental noises, so that a relatively uncontaminated signal can be obtained.

Signal processing schemes were used to treat AE signal to extract the most useful information, for example: time series analysis [4-7], fourier transform [8-10], gabor transform [11-13], wavelet transform [14-18], Dominant-Feature Identification [19], genetic programming [20], *etc.* Because the information obtained during the machining process is vague, incomplete or imprecise, these conventional methods need a large number of cutting experiments and additional assumptions in many circumstances for effective uncertainty handling. These requirements reduce the reliability of the models and make the implementation costly and time consuming. Moreover, the general mathematical relation cannot be used to map the nonlinear relation between the AE signal and tool wear condition [21].

This is not only due to the fact that the machining process varies considerably with part material, cutting fluids, chip formation, tool material, temperature, chatter and vibration, *etc.*, but also that that AE sensors are very sensitive to the environment change such as temperature, humidity, circuit noise and even the location error of the sensors. Moreover, changes of cutting conditions also affect the behaviour of acoustic emission signals. Artificial intelligence methods have played an important role in modern tool condition monitoring (TCM) to observe the relation between tool wear and AE signal such as neural networks [22-24], fuzzy system [25, 26] and fuzzy neural network [27-32], *etc.* The increased use of artificial intelligence within TCM has enabled the development of more robust and comprehensive strategies.

The aim of this paper is to present a Takagi-Sugeno-Kang (TSK) fuzzy system based on extended subtractive clustering method combined with the least square estimation method to identify AE signal in machining process. In this paper, the fuzzy identification method is used as a powerful tool to model highly complex non-linear physical processes and implemented to filter the raw AE signal directly from the AE sensor during high precision hard turning process.

This paper is divided into four sections. Section 4.1 is a brief introduction to previous studies on AE based TCM. The initial theoretical foundation, including TSK fuzzy logic system (FLS), subtractive clustering method and least-square estimation, and proposed modeling algorithm are in Section 4.2. An experimental case studying on fuzzy modeling on acoustic emission is presented in Section 4.3 following a discussion of the results. A conclusion is given in Section 4.4.

## 4.2 Subtractive clustering based fuzzy modeling

### 4.2.1 TSK fuzzy logic

Fuzzy logic was originally proposed by Zadeh in his famous paper “Fuzzy Sets” in 1965 [33]. Fuzzy logic provides a simple way to obtain a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information. TSK FLS [34, 35] was proposed in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and a mathematical function in the consequent part. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviour of the system in those regions. TSK FLS has a powerful capability of explaining complex relations among variables using rule consequents which are functions of the input variables. This is due to the model’s properties: on one hand being a general non-linear approximator that can approximate every continuous mapping and on the other hand being a piecewise linear model that is relatively easy to implement [36].

A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of the system. For a Multi-input-single -output (MISO) first-order type-1 TSK model, its  $k$ th rule can be expressed as:

IF  $x_1$  is  $Q_{1k}$  and  $x_2$  is  $Q_{2k}$  and ... and  $x_n$  is  $Q_{nk}$ ,

THEN  $Z$  is  $w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n$

where  $x_1, x_2, \dots, x_n$  and  $Z$  are linguistic variables;  $Q_{1k}, Q_{2k}, \dots$ , and  $Q_{nk}$  are the fuzzy sets on universe of discourses  $U, V, \dots$ , and  $W$ , and  $p_0^k, p_1^k, \dots$ , and  $p_n^k$  are regression parameters.

A Gaussian membership function (MF) can be expressed by the following formula for the  $v$ th variable:

$$Q_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*}}{\sigma} \right)^2 \right] \quad (4.1)$$

where  $x_v^{k*}$  is the mean of the  $v$ th input feature in the  $k$ th rule for  $v \in [0, n]$ .  $\sigma$  is the standard deviation of Gaussian MF.

The structure of a fuzzy TSK model can be done manually based on knowledge about the target process or using data-driven techniques. Identification of the system using clustering involves formation of clusters in the data space and translation of these clusters into TSK rules such that the model obtained is close to the system to be identified.

## 4.2.2 Extended Subtractive clustering

The aim of Chiu's subtractive clustering identification algorithm [37] is to estimate both the number and initial location of cluster centers and extract the TSK fuzzy rules from input/output data. Subtractive clustering operates by finding the optimal data point to define a cluster centre based on the density of surrounding data points. This method is a fast clustering method designed for high dimension problems with a moderate number of data points. This is because its computation grows linearly with the data dimension and as the square of the number of data points. A brief description of Chiu's subtractive clustering method is as follows:

Consider a collection of  $q$  data points  $\{x_1, x_2, \dots, x_n\}$  specified by  $m$ -dimensional  $x_j$ . Without loss of generality, assume the feature space is normalized so that all data are bounded by a unit hypercube. Calculate potential for each point by using equation below:

$$p_i = \sum_{j=1}^q e^{-\alpha \|x_i - x_j\|^2}, \quad \alpha = 4/r_a^2 \quad (4.2)$$

where  $\|.\|$  denotes the Euclidean distance. It is noted that only the fuzzy neighbourhood within the cluster radius  $r_a$  to the measure of potential.

After the potential of every data point has been computed, the data point with the highest potential is selected as the first cluster center. Assume  $x_1^*$  is the location of the first cluster center, and  $p_1^*$  is its potential value, then revise the potential of each data point  $x_j$  by the formula

$$p_i \Leftarrow p_i - p_1^* e^{-\beta \|x_i - x_1^*\|^2} \quad (4.3)$$

where  $\beta = 4/r_b^2$  and  $r_b = \eta r_a$  where  $\eta$  is squash factor

When the potential of all data points have been reduce by (4.3), the data point with the highest remaining potential is selected as the second cluster center. Then further reduce the potential of each data points. Generally, after  $k$ th cluster center has been obtained, the potential of each data point is revised by formula

$$p_i \Leftarrow p_i - p_k^* e^{-\beta \|x_i - x_k^*\|^2} \quad (4.4)$$

where  $x_1^*$  is the location of the  $k$ th cluster center and  $p_1^*$  is its potential value.

The process of acquiring new cluster center and revising potential repeats by using the following criteria:

if  $p_k^* > \varepsilon p_1^*$  ( $\varepsilon$  is accept ratio)

accept  $x_k^*$  as a cluster center and continue.

else if  $p_k^* < \varepsilon p_1^*$  ( $\varepsilon$  is reject ratio)

reject  $x_k^*$  and end the clustering process.

else

let  $d_{\min}$  = shortest of the distances between  $x_k^*$  and all previously found cluster centers.

if  $\frac{d_{\min}}{r_a} + \frac{p_k^*}{p_1^*} \geq 1$

accept  $x_k^*$  as a cluster center and continue.

else

reject  $x_k^*$  and set the potential at  $x_k^*$  to 0. Select the data point with the next highest potential as the new  $x_k^*$  and reset.

A standard Gaussian membership degree can be expressed as:

$$\mu_k = e^{-\alpha \|x - x_k^*\|^2} \quad (4.5)$$

Cluster center found in the training data are points in the feature space whose neighbourhood maps into the given class. Each cluster center can be translated into a fuzzy rule for identifying the class.

With subtractive clustering method,  $x_j$  is the  $j$ th input feature  $x_j$  ( $j \in [1, n]$ ), and  $Q_{jk}$  is the MF in the  $k$ th rule associated with  $j$ th input feature. The MF  $Q_{jk}$  can be obtained as

$$Q_{jk} = \exp \left[ -\frac{1}{2} \left( \frac{x_j - x_{jk}^*}{\sigma} \right)^2 \right] \quad (4.6)$$

where  $x_{jk}^*$  is the  $j$ th input feature of  $x_k^*$ , the standard deviation of Gaussian MF given as

$$\sigma = \sqrt{1/2\alpha} \quad (4.7)$$

The consequence parameter values can be obtained from a least squares estimation algorithm.

Subtractive clustering algorithm has various parameters to be set. Not knowing the best parameters to be used for a given data, a parameter search is performed to identify a better model. In his paper [38], Demirli described in detail the influences of the four parameters on clusters and fuzzy models, and proposed an extended subtractive clustering method with parametric search on various clustering parameters to identify the best model. As a result of parametric search, ranges of clustering parameters that provide best models are also identified (Table 4.1)

Table 4.1 Recommended value for parameters in subtractive clustering

Symbol	Chui	Demirli
Cluster radius	[0.25, 0.50]	[0.15, 1]
Squash factor	1.25	[0.05, 2]
Reject ratio	0.15	[0, 0.9]
Accept ratio	0.5	[0, 1]



### 4.2.3 Least Square Estimation Fuzzy modeling algorithm

For the first order model presented in this paper, the consequent functions are linear. In the method of Sugeno and Kang [35], least-square estimation is used to identify the consequent parameters of the TSK model, where the premise structure, premise parameters, consequent structure, and consequent parameters were identified and adjusted recursively. In a TSK FLS, rule premises are represented by an exponential membership function. The optimal consequent parameters  $p_0^k, p_1^k, p_2^k, \dots, p_n^k$  (coefficients of the polynomial function) for a given set of clusters are obtained using the least-square estimation method.

When certain input values  $x_1^0, x_2^0, \dots, x_n^0$  are given to the input variables  $x_1, x_2, \dots, x_n$ , the conclusion from the  $k$ th rule (1) in a TSK model is a crisp value  $w^k$ :

$$w^k = p_0^k + p_1^k x_1^0 + p_2^k x_2^0 + \dots + p_n^k x_n^0 \quad (4.8)$$

with a certain rule firing strength (weight) defined as

$$\alpha^k = \mu_1^k(x_1^0) \cap \mu_2^k(x_2^0) \cap \dots \cap \mu_n^k(x_n^0), \quad (4.9)$$

where  $\mu_1^k(x_1^0), \mu_2^k(x_2^0), \dots, \mu_n^k(x_n^0)$  are membership grades for fuzzy sets  $Q_1^k, Q_2^k, \dots$ , and  $Q_n^k$  in the  $k$ th rule. The symbol  $\cap$  is a conjunction operator, which is a T-norm (the minimum operator  $\wedge$  or the product operator  $*$ ).

Moreover, the output of the model is computed (using *weighted average aggregation*) as

$$w = \frac{\sum_{k=1}^m \alpha^k w^k}{\sum_{k=1}^m \alpha^k} \quad (4.10)$$

Suppose

$$\beta^k = \frac{\alpha^k}{\sum_{k=1}^m \alpha^k} \quad (4.11)$$

Then, (4.10) can be converted into a linear least-square estimation problem, as

$$w = \sum_{k=1}^m \beta^k w^k \quad (4.12)$$

For a group of  $\lambda$  data vectors, the equations can be obtained as

$$\begin{aligned}
w^1 &= \beta_1^1(p_0^1 + p_1^1 x_1 + p_2^1 x_2 + \dots + p_n^1 x_n) + \beta_1^2(p_0^2 + p_1^2 x_1 + p_2^2 x_2 + \dots + p_n^2 x_n) + \dots \\
&\quad + \beta_1^m(p_0^m + p_1^m x_1 + p_2^m x_2 + \dots + p_n^m x_n) \\
w^2 &= \beta_2^1(p_0^1 + p_1^1 x_1 + p_2^1 x_2 + \dots + p_n^1 x_n) + \beta_2^2(p_0^2 + p_1^2 x_1 + p_2^2 x_2 + \dots + p_n^2 x_n) + \dots \\
&\quad + \beta_2^m(p_0^m + p_1^m x_1 + p_2^m x_2 + \dots + p_n^m x_n) \\
&\quad \vdots \\
w^\lambda &= \beta_\lambda^1(p_0^1 + p_1^1 x_1 + p_2^1 x_2 + \dots + p_n^1 x_n) + \beta_\lambda^2(p_0^2 + p_1^2 x_1 + p_2^2 x_2 + \dots + p_n^2 x_n) + \dots \\
&\quad + \beta_\lambda^m(p_0^m + p_1^m x_1 + p_2^m x_2 + \dots + p_n^m x_n)
\end{aligned} \tag{4.13}$$

These equations can be represented as:

$$\begin{bmatrix}
\beta_1^1 x_1 & \beta_1^1 x_2 & \dots & \beta_1^1 x_n & \beta_1^1 & \cdot & \cdot & \cdot & \beta_1^m x_1 & \beta_1^m x_2 & \dots & \beta_1^m x_n & \beta_1^m \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\beta_\lambda^1 x_1 & \beta_\lambda^1 x_2 & \dots & \beta_\lambda^1 x_n & \beta_\lambda^1 & \cdot & \cdot & \cdot & \beta_\lambda^m x_1 & \beta_\lambda^m x_2 & \dots & \beta_\lambda^m x_n & \beta_\lambda^m
\end{bmatrix}
\begin{bmatrix}
p_1^1 \\
p_2^1 \\
\vdots \\
p_n^1 \\
p_0^1 \\
\cdot \\
\cdot \\
\cdot \\
p_1^m \\
p_2^m \\
\vdots \\
p_n^m \\
p_0^m
\end{bmatrix}
=
\begin{bmatrix}
w^1 \\
w^2 \\
\cdot \\
w^m
\end{bmatrix} \tag{4.14}$$

Using the standard notation  $AP = W$ , this becomes a least square estimation problem where  $A$  is a constant matrix (known),  $W$  is a matrix of output values (known) and  $P$  is a matrix of parameters to be estimated. The pseudo-inverse solution that minimizes  $\|AP - W\|^2$  is given by

$$P = (A^T A)^{-1} A^T W \tag{4.15}$$

#### 4.2.4 Fuzzy modeling algorithm

The diagram of fuzzy modeling algorithm is shown in Fig. 4.2. This algorithm is presented in [39, 40]. Subtractive clustering method combined with a least-square estimation algorithm is used to cope with the nonlinearity of the AE signal and the measurement data uncertainty.

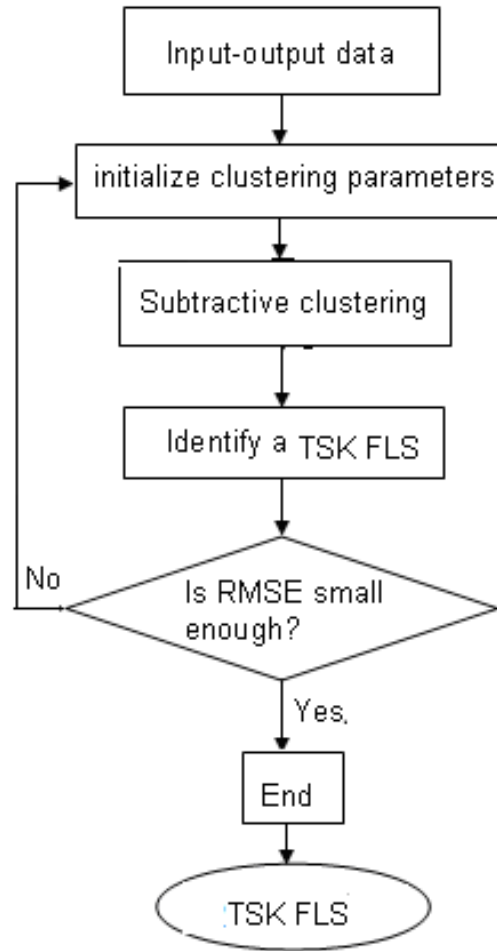


Fig. 4.2 Subtractive clustering based fuzzy approach

## 4.3 Case study

### 4.3.1 Experimental setup

The experiment of this paper was taken on the machine BOEHRINGER CNC Lathe shown in Fig. 4.3. The workpiece material is titanium metal matrix composite (Ti MMC) 10% wt. TiC/Ti-6Al-4V where the microstructural response of cast Ti-6Al-4V based composite contains 10 vol.-% TiC reinforcement. This kind of material is widely used in aerospace and military applications for its high hardness, light weight, high bending strength, fracture toughness, higher modulus, and elevated temperature resistance and high wear resistance. Consequently, its machining is very difficult.



Fig. 4.3 The BOEHRINGER CNC Lathe

The cutting tool insert was carbide from SECO tools (CNMG 120408 MF1 CP200). Turning test was done on a cylinder of Ti MMC 2.5" diameter in dry machining conditions. The machining speed was chosen to exceed the manufacturer's recommendation in order to check the tool cutting speed limits (80 m/min). The cutting depth was kept at 0.15 mm and the cutting feed was 0.1 mm.

Figure 4.4 shows the raw AE signal  $AE_{raw}$  directly from AE sensor. From Fig. 4.4, it is observed that the cutting tool leaves the surface of workpiece after 32 s. During the first 0~8 s, the cutting tool is approaching the workpiece and gradually reaching the cutting depth. The middle period (8~27s) is the steady cutting period, which contents the main useful information for tool wear condition investigation. This paper focuses on obtaining the best AE model during the continuous cutting periods using fuzzy modeling. The used part of  $AE_{raw}$  is shown in Fig. 4.5. TSK Fuzzy modeling is used to eliminate noise components in the AE signal.

### 4.3.2 Data processing

The clustering parameters are pre-initialized. The cluster radius  $r_a$  is confined to the range [0.15; 1.0] with a step size of 0.15. The accept ratio  $\bar{\varepsilon}$  and the reject ratio  $\underline{\varepsilon}$  are both considered in the

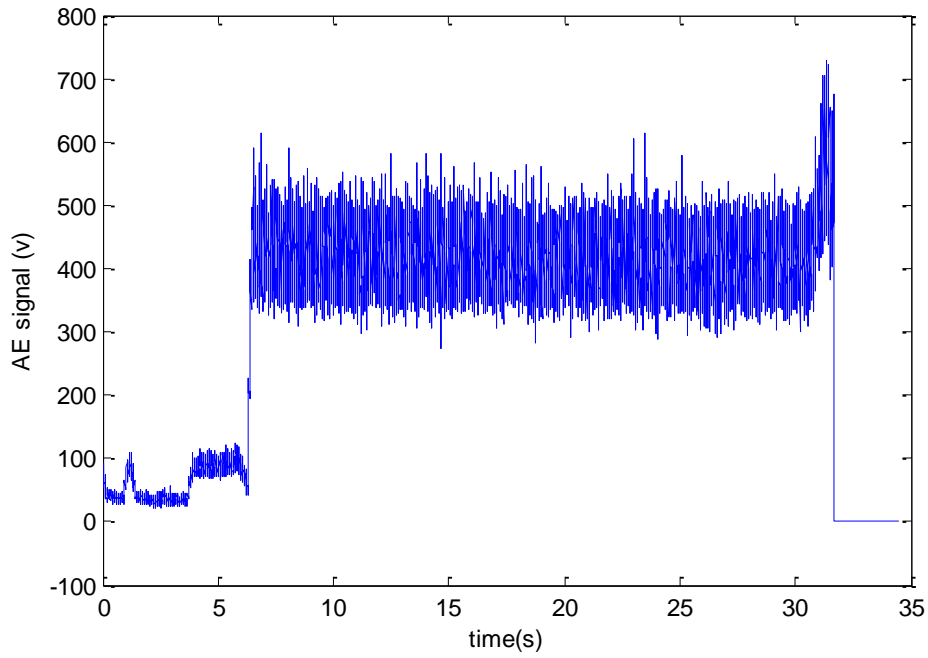


Fig. 4.4 Raw AE signal  $AE_{raw}$  directly from AE sensor

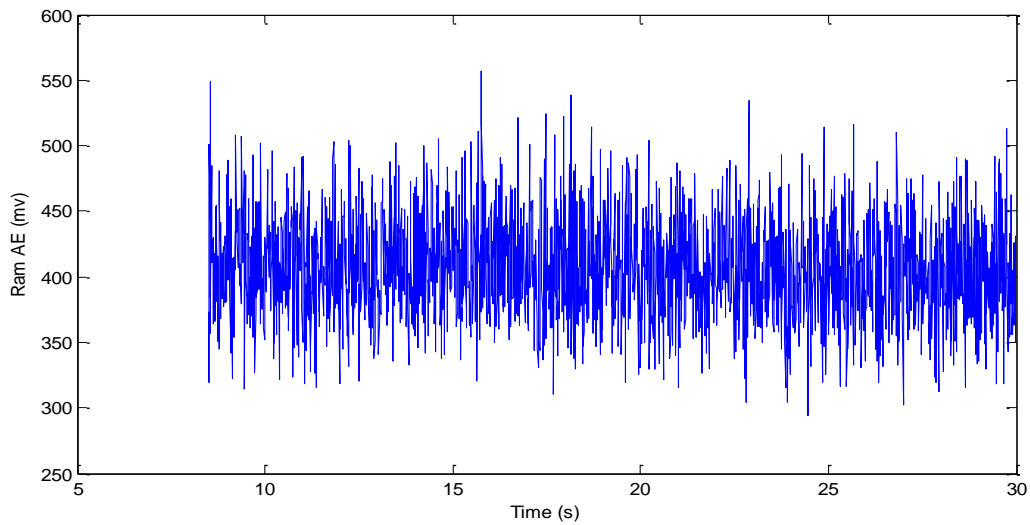


Fig. 4.5 AE signal used for data processing

range  $[0; 1.0]$  with a step size of 0.1. The squash factor  $\eta$  is considered in the range  $[0.05; 2]$  with a step size of 0.05. Combined with a least-square estimation algorithm, the fuzzy systems is identified.

Table 4.2 lists the identification parameters. In this experiment, 4800 data sets are used for the fuzzy modeling. Subtractive clustering parameters are given as 0.15 (cluster radius), 0.5 (Accept ratio), 0.15 (Reject ratio) and 0.1 (Squash factor). 23 clusters are listed in Table 4.2.

23 fuzzy rules are obtained and standard deviation of the identified Gaussian MFs is 0.7425. The 23 fuzzy premises are shown in Fig. 4.6 and modeling results is in Fig. 4.7. One example of these 23 fuzzy rules is expressed as the following:

$$\text{If } X_t = \left( -\frac{4}{(0.15)^2} \left( \frac{t - 25.6490}{0.7425} \right)^2 \right),$$

$$\text{then } Z = 5.954 \times 10^8 \times t - 1.198 \times 10^{11}$$

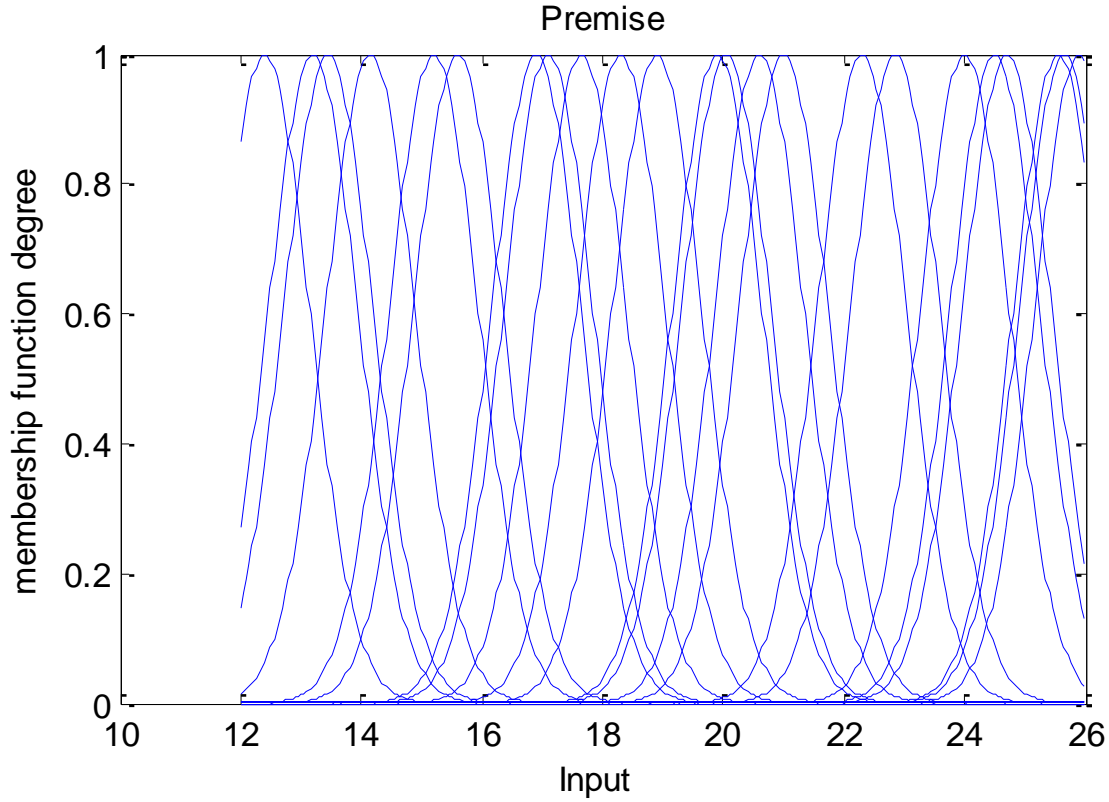


Fig. 4.6 Identified premise membership functions

Table 4.2 List of clustering parameters and identification results

Number of data sets	4800
Cluster radius	0.15
Accept ratio	0.5
Reject ratio	0.15
Squash factor	0.1
Number of rules	23
Standard deviation	0.7425
Cluster centers	22.2990 415.3226 15.5990 392.7174 18.8990 404.1768 13.1990 417.9160 24.6990 399.0800 16.8990 447.1252 20.9990 387.4298 14.1490 442.7461 17.6490 378.6971 20.0490 444.2090 13.4490 365.8690 22.8490 368.7549 24.4990 443.0127 17.0990 415.4027 18.2990 465.2846 25.5490 369.9846 25.8990 424.3272 19.8990 362.0750 15.1990 346.3021 12.3990 455.5406 23.9990 339.2642 20.5990 476.1064 25.6490 471.7862

Traditionally, the AE signal is characterized using AE root-mean-square (RMS) measurement in well-controlled tensile tests. To compare AE signal obtained by fuzzy filtering with the one by traditional filter, AE RMS values (illustrated in Fig. 4.8) and AE mean values (depicted in Fig. 4.9) are calculated for both cases. The formulas used to calculate the AE RMS and mean values are given by the following equations:

$$RMS = \sqrt{\left(\sum_{i=1}^n AE^2\right)/n} \quad (4.16)$$

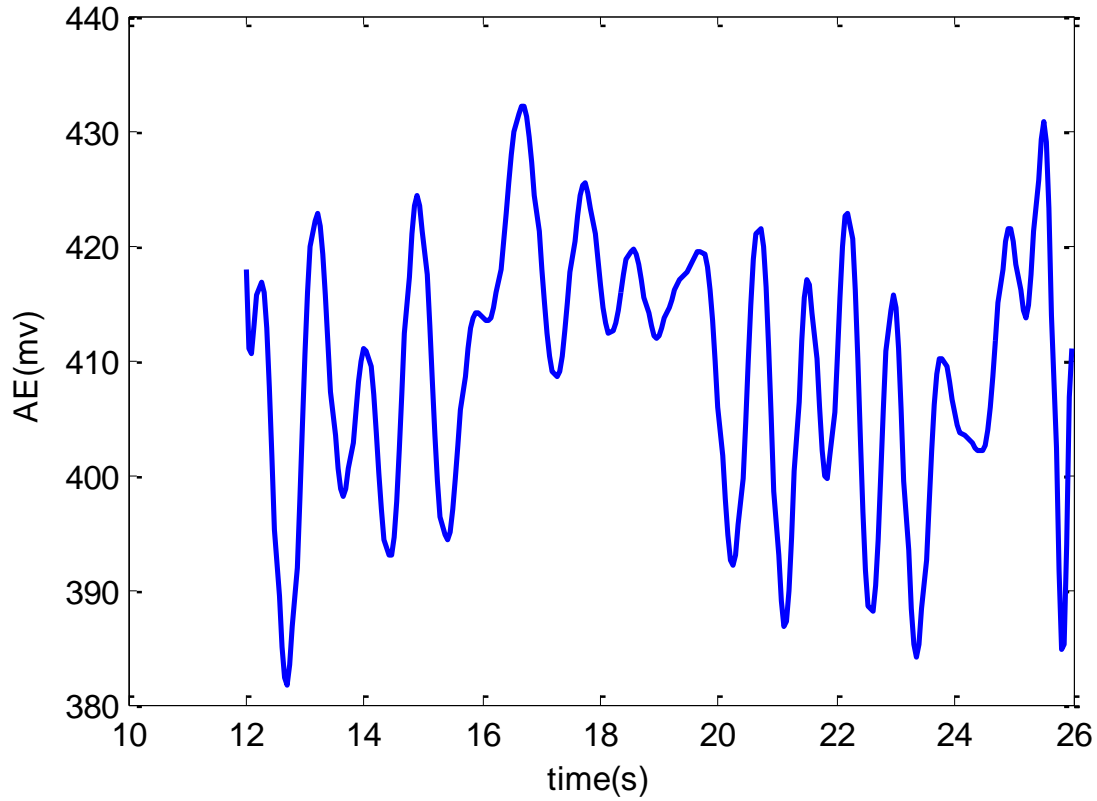


Fig. 4.7 AE after fuzzy identification

$$mean = \sum_{i=1}^n AE / n \quad (4.17)$$

The dotted curves (black) represent the values obtained by traditional filtering, and the solid ones (red) represent the values obtained by fuzzy filtering. In Fig. 4.8, the dotted curves are above the solid curves. That means the AE RMS values obtained by traditional filter are little higher than those by fuzzy filtering. The difference could be caused by different computation algorithm, where fuzzy filtering generates fuzzy rules directly from the input-output data acquired from sensor, without traditional user-defined high-pass and low-pass filters. The mean values obtained by the two methods are almost the same as shown in Fig. 4.9. During the cutting period from 14 -26 s, the average differences of the AE RMS and mean value are 1.4641mv and 0.1484mv , the maximums are 2.0954mv and 0.5195mv, also the minimums are 0.6192mv and 0.



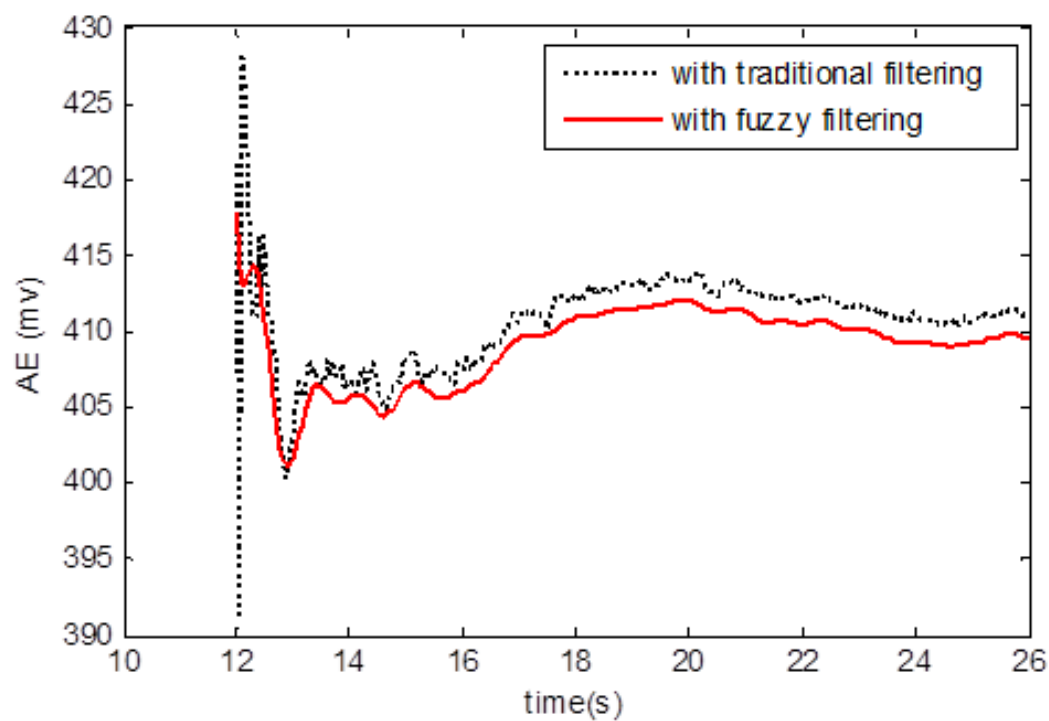


Fig. 4.8 AE RMS value

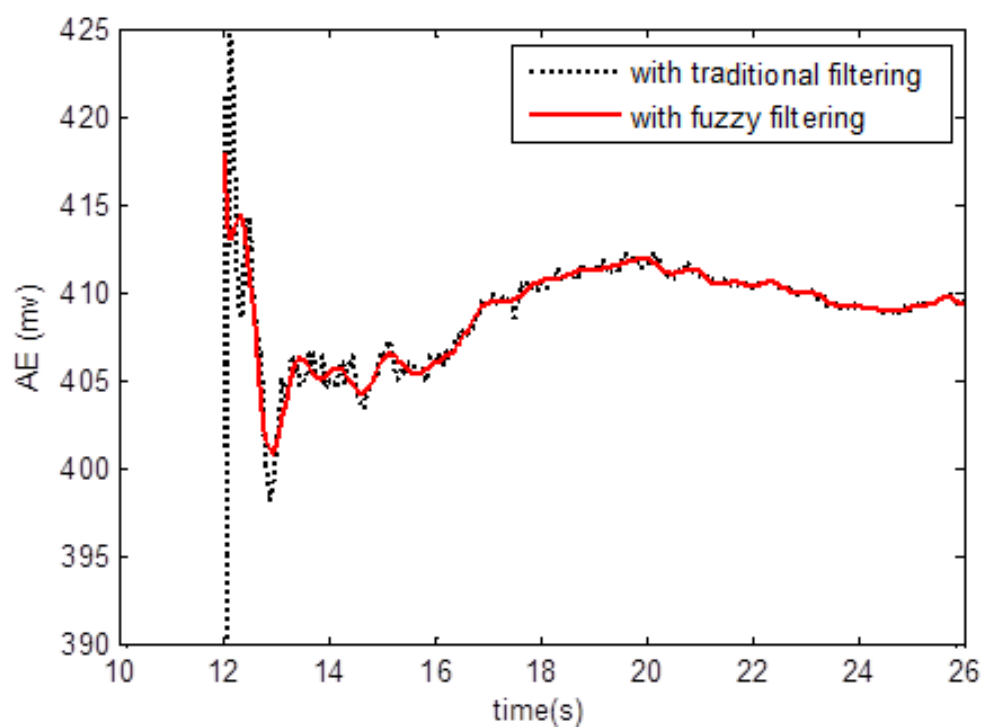


Fig. 4.9 AE mean value

## 4.4 Conclusion

This paper presented subtractive clustering based fuzzy identification method to determine AE signal in high precision hard turning process, combined with least-square estimation method. The comparison of the experimental results with that using traditional method shows that the fuzzy identification method can be used as a powerful tool to model highly complex non-linear physical processes and implemented to filter the raw AE signal directly from the AE sensor during high precision hard turning process with the difficulty in understanding the exact physics of the machining process. The fuzzy modelling is efficient and feasible.

The limitation of this paper is that the experimental study is limited to filtering an modelling one AE signal. Continue study on multiple data sets can be done to obtain the best model cutting AE during machining process and effectively use the fuzzy model for tool condition monitoring applications.

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## Reference

- [1] Ren, Q. Baron L. and Balazinski, M.: Application of Type-2 Fuzzy Estimation on Uncertainty in Machining: An Approach on Acoustic Emission during Turning Process. The 28th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2009), Cincinnati, Ohio, USA (2009)
- [2] Wang, L. H., Gao. R. X.: Condition Monitoring and Control for Intelligent Manufacturing, Springer Series in Advanced Manufacturing, ISBN: 978-1-84628-268-3 (2006)
- [3] Iwata, K., Moriwaki, T.: Application of acoustic emission measurement to in-process sensing of tool wear, Ann. CIRP, 26(1-2) 19–23 (1977)
- [4] Liang, S., Dornfeld, D.: Tool wear detection using time series analysis of acoustic emission. J. Eng. Ind. Trans. ASME.111(3) 199-205 ( 1989)

- [5] Marinescu, I., Axinte, D.: A time-frequency acoustic emission-based monitoring technique to identify workpiece surface malfunctions in milling with multiple teeth cutting simultaneously. *International Journal of Machine Tools and Manufacture*, 49(1) 53-65 (2009)
- [6] Polito F., Petri A., Pontuale G. and Dalton F.: Analysis of metal cutting acoustic emissions by time series models. *International Journal of Advanced Manufacturing Technology*, 48 (9-12) 897-903 (2010)
- [7] Vavourakis, V., Protopappas, V.I.C., Fotiadis, D. and Polyzos, D.: Numerical determination of modal dispersion and AE signal characterization in waveguides through LBIE/BEM and time-frequency analysis. *Computational Mechanics* 43(3) 431-441 (2009)
- [8] Emel, E. and Kannatey-Asibu, Jr. E.: Tool failure monitoring in turning by pattern recognition analysis of AE signal. *Trans. ASME J. Eng. Ind.* 110 137–145 (1988)
- [9] Li, X. and Yuan, Z.: Tool wear monitoring with wavelet packet transform-fuzzy clustering method. *Wear* 219(2) 145–154 (1998)
- [10] Ince N. F., Kao Chu-Shu, Kaveh M., Tewfik A. and Labuz J. F.: A Machine Learning Approach for Locating Acoustic Emission, *EURASIP Journal on Advances in Signal Processing* 2010(895486), doi:10.1155/2010/895486 (2010)
- [11] Menon, A. and Boutaghou, Z.: Time-frequency analysis of tribological systems—Part I: implementation and interpretation. *Tribol. Int.* 31(9) 501–510 (1998)
- [12] Heil, C. and Walnut, D.: Continuous and discrete wavelet transforms, *SIAM Rev.* 31(4), 628–666 (1989)
- [13] Serrano, E. and Fabio, M.: Application of the wavelet transform to acoustic emission signals processing. *IEEE Trans. Signal Proces* 44(5) 1270–1275 (1996)
- [14] Kamarthi, S., Kumara, S. and Cohen P.: Wavelet representation of acoustic emission in turning process. *Intell. Eng. Syst. Artif. Neural Netw.* 5 861–866 (1995)
- [15] Li, X: Real-time detection of the breakage of small diameter drills with wavelet transform. *Int. J. Adv. Manuf. Technol.* 14(8), 539–543 (1998)
- [16] Van Dijck, G., Wevers M.; Van Hulle, M.: Wavelet packet decomposition for the identification of corrosion type from acoustic emission signals. *International Journal of Wavelets, Multiresolution and Information Processing*, 7(4) 513-534 (2009)
- [17] Kasashima, N., Mori K., Herrera Ruiz G. and Taniguchi N.: Online failure detection in face

- milling using discrete wavelet transform. *Ann. CIRP* 44(1) 483–487 (1995)
- [18] Kious M., Ouahabi A., Boudraa M., Serra R. and Cheknane A.: Detection process approach of tool wear in high speed milling. *Measurement* 43(10) 1439-1446 (2010)
- [19] Zhou J. H., Pang C. K., Zhong Z W. and F. L. Lewis: Tool wear monitoring using acoustic emissions by dominant-feature identification. *IEEE Transactions on Instrumentation and Measurement* 60(2) 547-559 (2011)
- [20] Griffin, J. M. and Chen, X.: Multiple classification of the acoustic emission signals extracted during burn and chatter anomalies using genetic programming, *International Journal of Advanced Manufacturing Technology* 45(11-12) 1152-1168 (2009)
- [21] Li, X: A brief review: acoustic emission method for tool wear monitoring during turning. *International Journal of Machine Tools and Manufacture* 42(2) 157-165 (2002)
- [22] Darenfed, S. and Wu, S.: Polynomial learning networks for cutting tool diagnosis in machining operations. *Trans. Can. Soc. Mech. Eng.* 16(2) 147–163 (1992)
- [23] Quan Y., Zhou, M. and Luo, Z.: On-line robust identification of tool-wear via multi-sensor neural-network fusion. *Engineering Applications of Artificial Intelligence* 11(6) 717-722 (1998)
- [24] Lee, S. H. and Lee, D.: In-process monitoring of drilling burr formation using acoustic emission and a wavelet-based artificial neural network. *International Journal of Production Research*, 46(17) 4871-4888 (2008)
- [25] Ren, Q., Baron, L. and Balazinski, M.: Type-2 TSK Fuzzy Filtering and Uncertainty Estimation for Acoustic Emission in Precision Manufacturing. *Proceedings of the IASTED International Conference Computational Intelligence (IASTED-CI 2009)*, Honolulu, Hawaii, USA 170-175 (2009)
- [26] Ren Q., Baron L., and Balazinski M.: Type-2 Fuzzy Modeling for Acoustic Emission Signal in Precision Manufacturing, *Modeling and Simulation in Engineering*, article in press (2011)
- [27] Li, Y. Y. and Yuan, Z.: On-line tool condition monitoring system with wavelet fuzzy neural network. *Journal of Intelligent Manufacturing* 8 271-276 (1997)
- [28] Kuo, R. J. and Cohen, P. H.: Intelligent tool wear estimation system through artifices neural networks and fuzzy modeling. *Artificial Intelligence in Engineering*, vol. 12, no.3, pp. 229-242, 1998
- [29] Lezanski, P.: An intelligent system for grinding wheel condition monitoring. *Journal of*

- Materials Processing Technology 109(3) 258-263 (2001)
- [30]Sun J., Rahman, M., Wong Y. S. and Hong, G. S.: Multiclassification of tool wear with support vector machine by manufacturing loss consideration. International Journal of Machine Tools and Manufacture 44(11) 1179-1187 (2004)
- [31]Deiab, I., Assaleh, K. and Hammad, F.: On modeling of tool wear using sensor fusion and polynomial classifiers. Mechanical Systems and Signal Processing, 23(5) 1719-1729 (2009)
- [32]Malekian, M., Park, S. S. and Jun, M. B.G: Tool wear monitoring of micro-milling operations. Journal of Materials Processing Technology 209(10) 4903-4914 (2009)
- [33]Zadeh, L. A.: Fuzzy Sets. Information and Control 8 338-353 (1965)
- [34]Takagi, T. and Sugeno, M.; Fuzzy Identification of Systems and Its Applications to Modeling and Control. IEEE Transaction on Systems, Man, and Cybernetics 15(1) 116-132 (1985)
- [35]Sugeno, M. and Kang, G.: Structure Identification of Fuzzy Model. Fuzzy Sets and Systems 28(1) 15-33 (1988)
- [36]Johansen, T. and Foss, B.: Identification of Non-Linear System Structure and Parameters Using Regime Decomposition. Automatic 31(2) 321–326 (1995)
- [37]Chiu, S. L.: Fuzzy Model Identification Based on Cluster Estimation. J. on Int. Fuzzy Sys. 2 267–278 (1994)
- [38]Demirli, K., Cheng, S. X. and Muthukumaran, P.: Subtractive clustering based on modelling of job sequencing with parametric search. Fuzzy Sets and Systems 137 235–270 (2003)
- [39]Ren, Q., Baron, L., Balazinski, M. and Jemelniak, K.: Tool Condition Monitoring Using the TSK Fuzzy Approach Based on Subtractive Clustering Method. News Frontiers in Applied Artificial Intelligence 52-61 Springer Berlin (2008)
- [40]Ren, Q., Baron, L., Balazinski, M. and Jemelniak, K.: TSK fuzzy modeling for tool wear condition in turning processes: An experimental study. Engineering Applications of Artificial Intelligence 24(2) 260-265 (2011)

## CHAPTER 5      TYPE-2 FUZZY MODELING FOR ACOUSTIC EMISSION SIGNAL IN PRECISION MANUFACTURING

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**Abstract** – This paper presents an application of type-2 fuzzy logic on acoustic emission (AE) signal modeling in precision manufacturing. Type-2 fuzzy modeling is used to identify the AE signal in precision machining. It provides a simple way to arrive at a definite conclusion without understanding the exact physics of the machining process. Moreover, the interval set of the output from the type-2 fuzzy approach assesses the information about the uncertainty in the AE signal, which can be of great value for investigation of tool wear conditions. Experiments show that the development of the AE signal uncertainty trend corresponds to that of the tool wear. Information from the AE uncertainty scheme can be used to make decisions or investigate the tool condition so as to enhance the reliability of tool wear.

**Key words** – fuzzy modeling, subtractive clustering, precision machining, acoustic emission, tool wear

### 5.1 Introduction

Related to advances in machine tools, manufacturing systems and material technology, machining practice is advancing from conventional machining to precision machining, even high

precision machining. The scale of precision machining becomes finer and closer to the dimensional scale of material properties. As a result, the acoustic emission (AE) from microscopic sources becomes significant [1].

AE is the class of phenomena whereby transient elastic waves are generated by the rapid release of energy by a localized source or sources within a material, or transient elastic wave(s) so generated (ANSI/ASTM E 610-89). Emissions from process changes, like tool wear, chip formation, can be directly related to the mechanics of the process. AE-based sensing methodologies for tool condition and cutting process monitoring have been studied since 1977 [2]. Signal processing schemes were used to treat AE signal to extract the most useful information, for example: time series analysis [3, 4], Fourier transform [5, 6], Gabor transform [7-9] and wavelet transform [10-13], *etc.* Because the information obtained during the machining process is vague, incomplete or imprecise, these conventional methods need a large number of cutting experiments and additional assumptions in many circumstances for effective uncertainty handling. These requirements reduce the reliability of the models and increase money and time consumption. Moreover, the general mathematical relation cannot be used to map the nonlinear relationship between the AE signal and tool wear condition [14]. Artificial intelligence methods have played an important role in modern tool condition monitoring (TCM) to observe the relation between tool wear and AE signal such as neural networks [15, 16], fuzzy logic [17] and fuzzy neural network [18-22]. The increased use of artificial intelligence within TCM has enabled the development of more robust and comprehensive strategies.

It is believed that a relatively uncontaminated AE signal can be obtained because AE frequency range is much higher than that of machine vibrations and environmental noises and does not interfere with the cutting operation. AE can be effectively used for TCM applications at the precision scale. In fact, it is impossible to get an accurate AE signal. It is because the machining process varies considerably depending on the part material, temperature, cutting fluids, chip formation, the tool material, temperature, chatter and vibration, *etc.* Additionally, AE sensors are very sensitive to environmental changes such as changes in temperature, humidity, circuit noise, even the locating error of the sensors. Moreover, changes of cutting conditions also affect the behaviour of acoustic emission signals. None of previous studies considered the uncertainty in AE signal.

The aim of this paper is to present an innovative type-2 Takagi-Sugeno-Kang (TSK) fuzzy modeling to capture the uncertainties in the AE signal in machining process in order to overcome the challenges in TCM. Type-2 TSK fuzzy modeling method is not only a powerful tool to model high complex non-linear physical processes, but also a great estimator for the ambiguities and uncertainties associated with the system. It is capable to arrive at a definite conclusion without understanding the exact physics of the machining process. In this paper, type-2 TSK fuzzy modeling is implemented to filter the raw AE signal directly from the AE sensor during turning process. Furthermore, its output interval set assesses the uncertainty information in AE, which is of great value to a decision maker and can be used to investigate the complicated tool wear condition during machining process.

This paper is divided into four sections. Section 5.1 gives a brief overview of previous studies on AE based TCM. Section 5.2 introduces TSK fuzzy logic and the type-2 TSK fuzzy modeling algorithm. Section 5.3 presents a case study where type-2 TSK fuzzy approach is used to filter the raw AE signal directly from the AE sensor and identify the uncertainty interval of AE. The experimental results show the effectiveness and advantages of type-2 TSK fuzzy modeling. Conclusion is given in Section 5.4.

## **5.2 Type-2 TSK Fuzzy Uncertainty Modeling**

### **5.2.1 TSK fuzzy logic**

Fuzzy logic has been originally proposed by Zadeh in his famous paper “Fuzzy Sets” in 1965 [23]. Fuzzy logic provides a simple way to obtain a definite conclusion based upon vague, ambiguous, imprecise, noisy, or missing input information. TSK fuzzy logic system (FLS) [24, 25] was proposed in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and a mathematical function in the consequent part. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviour of the system in those regions. TSK FLS has a powerful capability of explaining complex relations among variables using rule consequents which are functions of the input variables. This is due to the model’s properties: on one hand being a general non-linear approximator that can approximate every continuous



mapping and on the other hand being a piecewise linear model that is relatively easy to interpret [26] and whose linear sub-models can be exploited for control and fault detection [27].

A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of the system. For a MISO first-order type-1 TSK model, its  $k$ th rule can be expressed as:

IF  $x_1$  is  $Q_{1k}$  and  $x_2$  is  $Q_{2k}$  and ... and  $x_n$  is  $Q_{nk}$ ,

THEN  $Z$  is  $w^k = p_0^k + p_1^k x_1 + p_2^k x_2 + \dots + p_n^k x_n$

where  $x_1, x_2, \dots, x_n$  and  $Z$  are linguistic variables;  $Q_{1k}, Q_{2k}, \dots$ , and  $Q_{nk}$  are the fuzzy sets on universe of discourses  $U, V, \dots$ , and  $W$ , and  $p_0^k, p_1^k, \dots$ , and  $p_n^k$  are regression parameters.

A Gaussian MF can be expressed by the following formula for the  $v$ th variable:

$$Q_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*}}{\sigma} \right)^2 \right] \quad (5.1)$$

where  $x_v^{k*}$  is the mean of the  $v$ th input feature in the  $k$ th rule for  $v \in [0, n]$ .  $\sigma$  is the standard deviation of Gaussian MF.

Based on Zadeh's concept of type-2 fuzzy sets and extension principal [28], practical algorithms for conjunction, disjunction and complement operations of type-2 fuzzy sets are obtained by extending previous studies [29]. Prior work [30] also introduced embedded interval-valued type-2 fuzzy sets and developed a general formula for the extended composition of type-2 relations, which is considered as an extension of the type-1 composition. The characterization in the definition of type-2 fuzzy sets uses the notion that type-1 fuzzy sets can be thought of as a first order approximation of uncertainty and, therefore, type-2 fuzzy sets provide a second order approximation. They play an important role in modeling uncertainties that exist in fuzzy logic systems [31], and are becoming increasingly important in the goal of "Computing with Words" [32] and "Computational Theory of Perceptions" [33]. A complete type-2 fuzzy logic theory with the handling of uncertainties was also established [34]. Because of its larger number of design parameters for each rule, it was believed that type-2 FLS have the potential to be used in control [35] and other areas where a type-1 model may be unable to perform well [36]. Type-2 TSK FLS and its structures were presented in 1999 [37].

An example of a type-2 MF, whose vertices have been assumed to vary over some interval of value, is depicted in Fig. 5.1. The *footprint of uncertainty* (FOU) associated with this type-2 MF is a bounded shaded region. FOU represents the entire interval type-2 fuzzy set  $\tilde{Q}$ . Upper MF and Lower MF are two type-1 MFs that are bounds for the FOU of a type-2 set  $\tilde{Q}$ . The intersections of crisp input  $x^0$  show that there are lower MF degree  $\underline{\mu}$  and upper MF degree  $\bar{\mu}$  with respective lower and upper MFs. Detailed type-2 fuzzy sets and interval type-2 FLS background material can be found in [38].

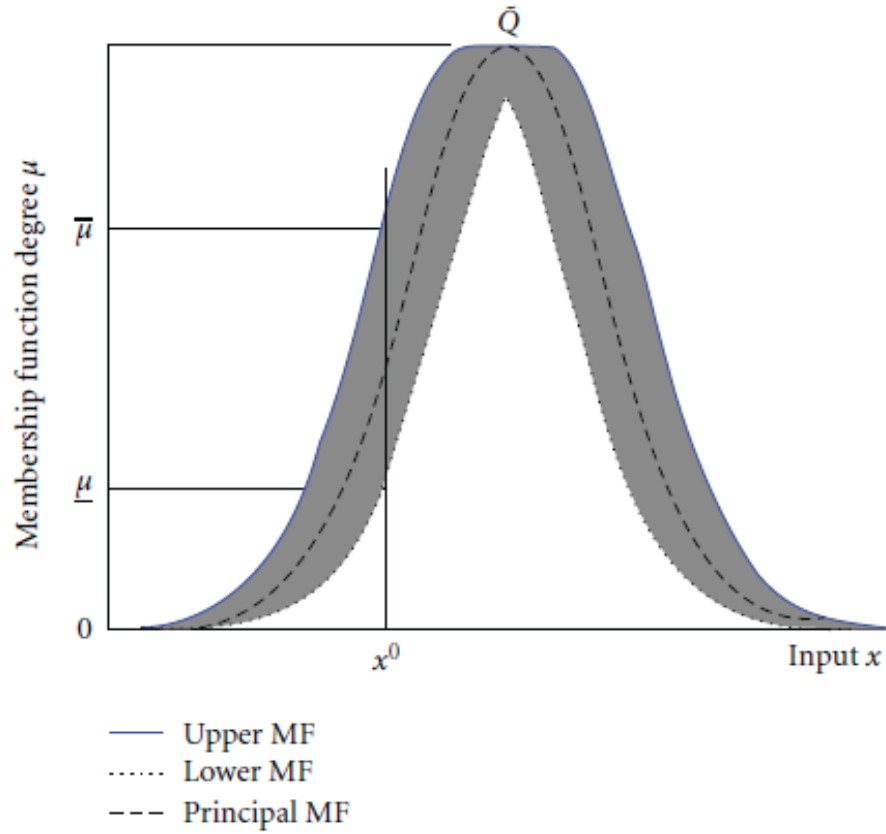


Fig. 5.1 Type-2 Gaussian MF

A generalized  $k$ th rule in the first-order type-2 TSK fuzzy MISO system can be expressed as

IF  $x_1$  is  $\tilde{Q}_{1k}$  and  $x_2$  is  $\tilde{Q}_{2k}$  and ... and  $x_n$  is  $\tilde{Q}_{nk}$ ,

THEN  $Z$  is  $\tilde{w}^k = p_0 + p_1 \tilde{x}_1 + p_2 \tilde{x}_2 + \dots + p_n \tilde{x}_n$

where  $\tilde{p}_0^k, \tilde{p}_1^k, \dots, \tilde{p}_n^k$  are consequent parameters,  $\tilde{w}^k$  is the output from the  $k$ th IF-THEN rule in an  $M$ -rule FLS,  $\tilde{Q}_{1k}, \tilde{Q}_{2k}, \dots, \tilde{Q}_{nk}$  are fuzzy sets on the universe of discourse.

One way to obtain a type-2 model directly from a type-1 model is by extending the cluster center,  $x_{jk}^*$ , from a point to a constant-width interval-valued fuzzy set,  $\tilde{x}_{jk}^*$  as shown in Fig. 5.2. The size of the interval is  $2a$ :

$$\tilde{x}_{jk}^* = [x_{jk}^* (1 - a_j^k), x_{jk}^* (1 + a_j^k)] \quad (5.2)$$

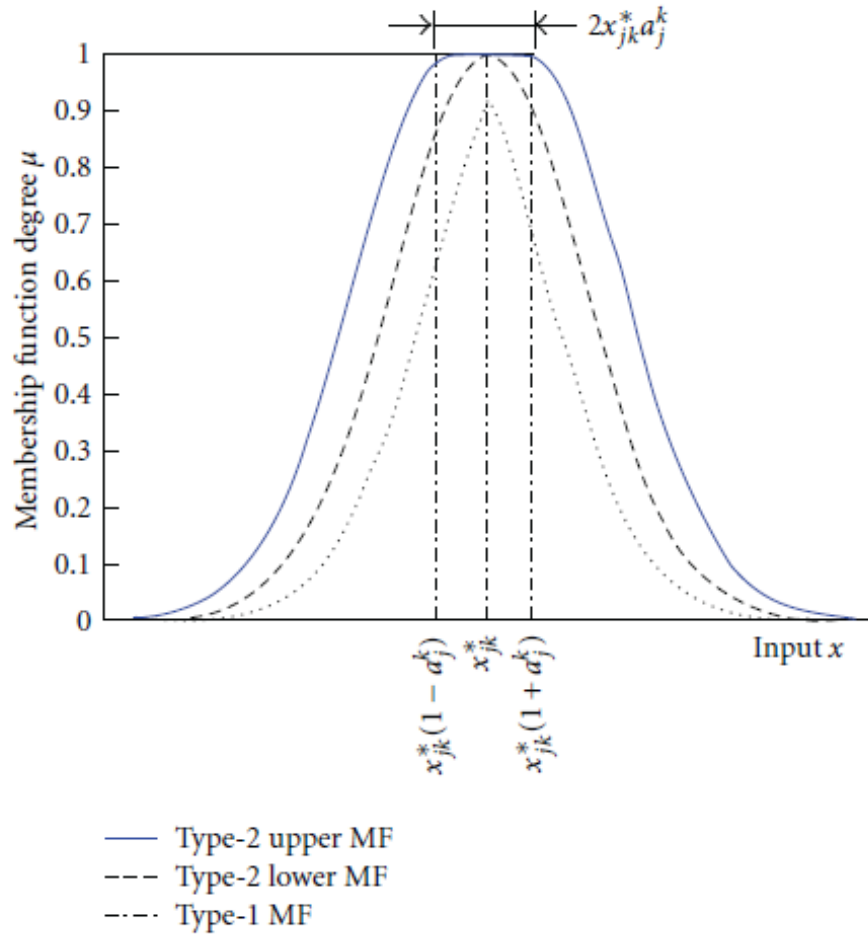


Fig. 5.2 Spread of cluster center

Consequent parameter  $\tilde{p}_j^k$  is obtained by extending the consequent parameter  $p_j^k$  from its type-1 counterpart using the following expression:

$$\tilde{p}_j^k = \begin{bmatrix} p_j^k - s_j^k, p_j^k + s_j^k \end{bmatrix}, \quad (5.3)$$

where  $j \in [0, n]$ , and  $s_j^k$  denotes the spread of fuzzy numbers  $p_j^k$ .

Hence, the premise MF is changed from type-1 fuzzy set into type-2 fuzzy set, *i.e.*,

$$\tilde{Q}_{jk} = \exp \left[ -\frac{1}{2} \left( \frac{x_j - x_{jk}^* (1 \pm a_j^k)}{\sigma_j^k} \right)^2 \right] \quad (5.4)$$

where  $\sigma_j^k$  is the standard deviation of Gaussian MF.

Type-2 FLSs are very useful in circumstances in which it is difficult to determine an exact membership function for a fuzzy set. They can be used to handle rule uncertainties and even measurement uncertainties. Type-2 FLSs moves the world of FLSs into a fundamentally new and important direction. To date, type-2 FL moves in progressive ways where type-1 FL is eventually replaced or supplemented by type-2 FL [39].

### 5.2.2 Type-2 fuzzy modelling algorithm

The diagram of type-2 TSK fuzzy modelling algorithm is shown in Fig. 5.3. This algorithm is initially presented in [40]. Type-2 TSK fuzzy approach includes two steps – The first step is type-1 fuzzy modelling to eliminate noise components in the AE signal and the second consists of expanding the type-1 fuzzy system to its type-2 counterpart to obtain the information of uncertainty in AE signal. The first step is type-1 TSK fuzzy approach. Subtractive clustering method [41] (see Appendix A) combined with a least-square estimation algorithm is used to cope with the nonlinearity of the AE and the uncertainty of imprecise data from measurement. A detailed description for this type-1 fuzzy modelling can be found in [42, 43]. The second step is type-2 TSK fuzzy approach. The type-1 MFs are considered as the principal MFs of a type-2 FLS, the antecedent MFs are extended as interval type-2 fuzzy memberships by assigning uncertainty to cluster centers using eq. (5.2), and the consequent parameters are extended as fuzzy numbers by assigning uncertainty to consequent parameter values using eq. (5.3). The type-2 TSK fuzzy inference engine is presented in Appendix B. Through enumerative search of

optimum values of spreading percentage of cluster centers and consequent parameters, the best approach for analysing AE signal is obtained. The Detailed description for this modelling algorithm can be found in [44,]. Examples of application of this algorithm can be found in [45-47].

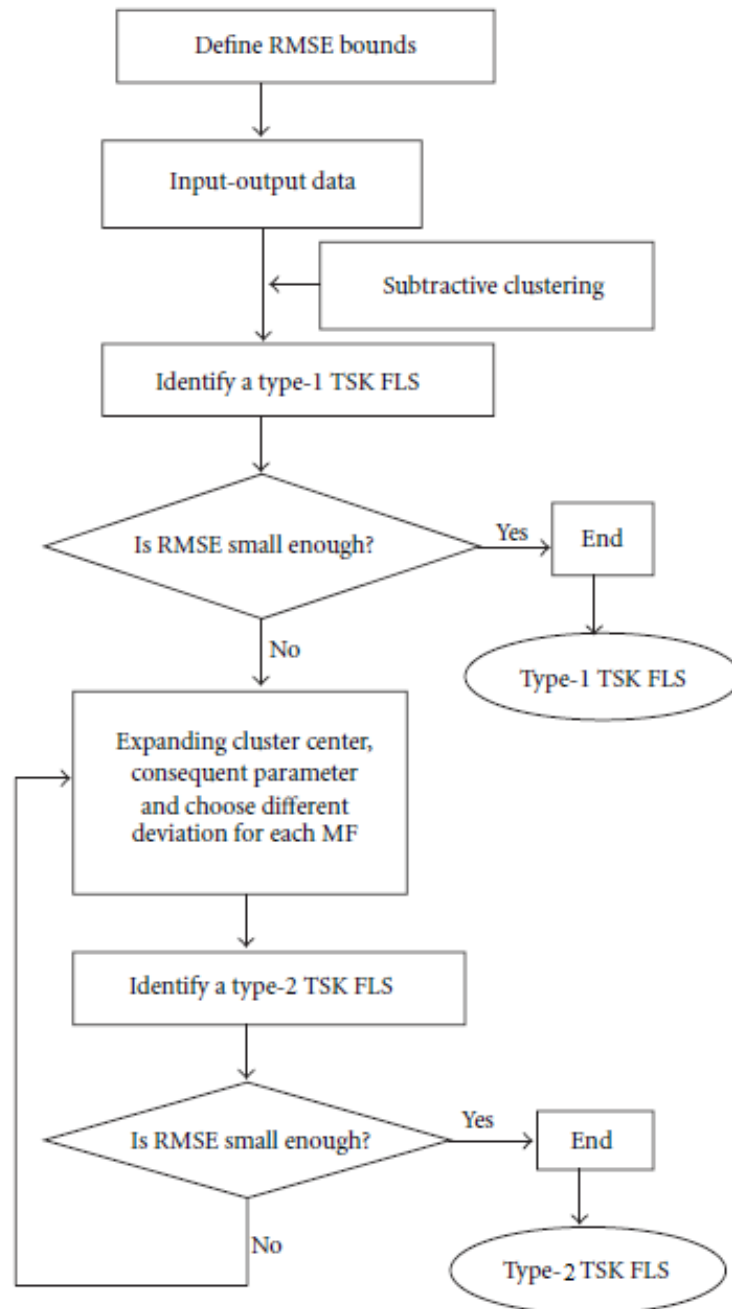


Fig. 5.3 Diagram of subtractive clustering based type-2 TSK fuzzy approach

Compared with traditional methods and its type-1 counterpart, type-2 fuzzy modeling can not only obtain a modeling result directly from the input-output data sets, but it can also capture the uncertainty interval of the result [34, 48]. The information about uncertainties in the type-2 interval output is very helpful for decision making.

## 5.3 Experimental study

### 5.3.1 Experimental setup

The experiment described in this paper was taken on the BOEHRINGER CNC lathe. The workpiece material was Titanium Metal matrix Composite (Ti MMC) 10% wt. TiC/Ti-6Al-4V where the microstructural response of cast Ti-6Al-4V based composite contains 10 vol.-% TiC reinforcement. This kind of material is widely used in aerospace and military applications for its high hardness, light weight, high bending strength, fracture toughness, higher modulus, and elevated temperature resistance and high wear resistance. Consequently, its machining is very difficult.

The cutting tool insert was carbide from SECO tools (CNMG 120408 MF1 CP200). Turning test was done on a cylinder of Ti MMC 2.5" diameter in dry machining conditions. The machining speed was chosen to exceed the manufacturer's recommendation in order to see and check the tool cutting speed limits (80 m/min). The cutting depth was kept at 0.15mm and the cutting feed was 0.1mm.

The aim of this study was to find out the relation between AE and tool wear. During the test, every time when cutting length reached 10mm, the machine was stopped to manually measure the tool wear parameter ( $VB_b$ ).

Figure 5.4 shows one example of a raw AE signal  $AE_{raw}$  directly from AE sensor. During the first 5~8s, the cutting tool is approaching the workpiece and gradually reaching the cutting depth. After 30s, the cutting tool leaves the surface of workpiece. The middle period is the steady cutting period, which contains the most useful information for tool wear condition investigation. In the experiment, five AE signal sets were recorded according to different cutting sections: 0 ~ 10 mm, 10 ~ 20 mm, 20 ~ 30 mm, 30 ~ 40 mm and 40 ~ 50 mm.

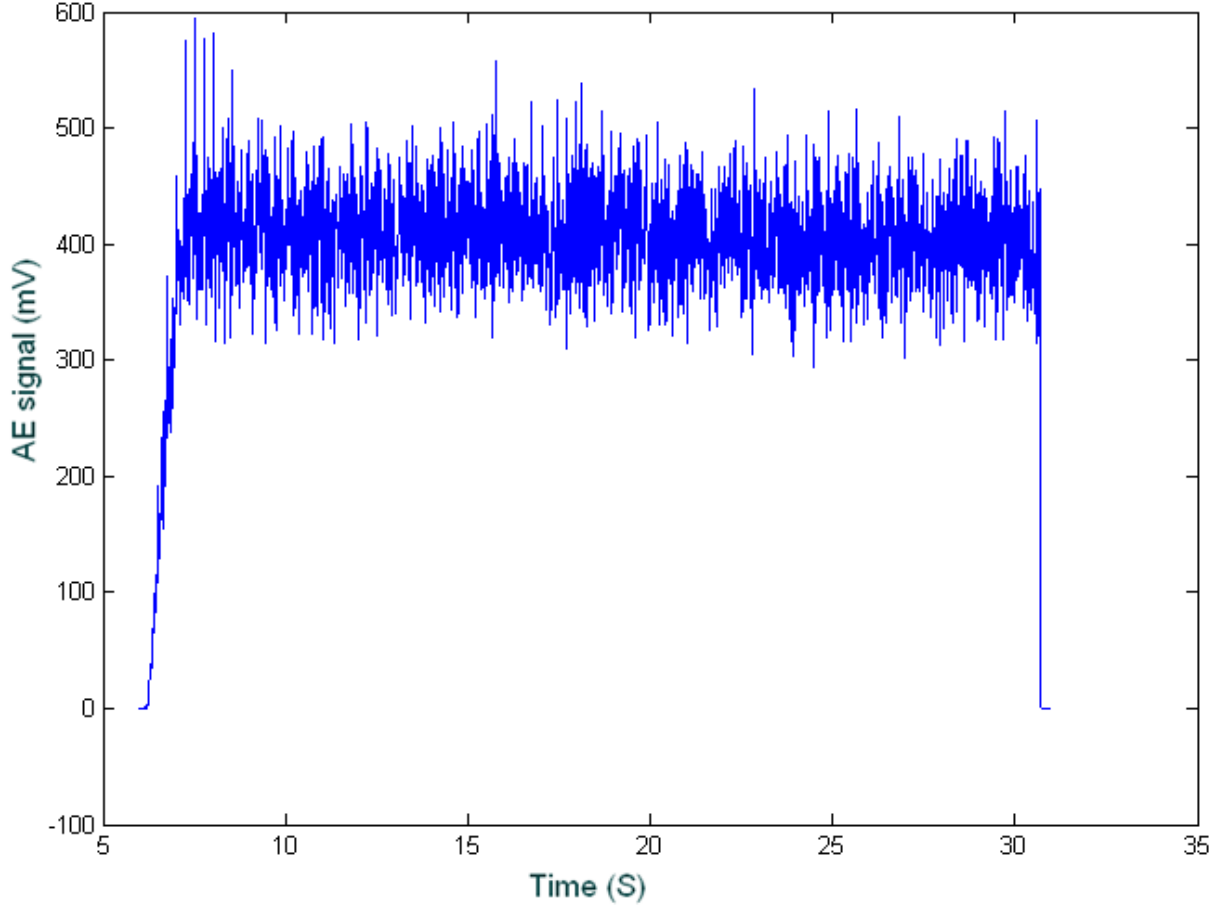


Fig. 5.4 AE signal from cutting process

### 5.3.2 Data processing

First, Type-1 TSK fuzzy filtering (top part of Fig. 5.3) was used to eliminate noise components in the AE signal. Demirli's extended subtractive clustering identification algorithm [49] was used to estimate both the number and initial location of cluster centers and extract the TSK fuzzy rules from input/output data. The clustering parameters are pre-initialized. The cluster radius is confined to the range  $[0.15; 1.0]$  with a step size of 0.15. The accept ratio and the reject ratio are both considered in the range  $[0; 1.0]$  with a step size of 0.1. The squash factor is considered in the range  $[0.05; 2]$  with a step size of 0.05. Combined with a least-square estimation algorithm, the fuzzy systems for each cutting length were identified. Table 5.1 lists the number of rules identified and the standard deviation used for the five AE signal sets. List of cluster centres can be found in [50].

Table 5.1 Number of rules and parameters of type-1 TSK modeling

Cutting section (mm)	0 ~ 10	10 ~ 20	20 ~ 30	30 ~ 40	40 ~ 50
Number of data sets	5500	8200	5700	4500	4000
Number of rules	24	23	29	34	22
Standard deviation	1.1209	0.9016	1.034	1.1932	1.0606

Traditionally, the AE signal is characterized using AE root-mean-square (RMS) measurement in well-controlled tensile tests. To compare AE signal obtained by fuzzy filtering with the one by traditional filter, AE RMS values (illustrated in Fig. 5.5) and AE mean values (depicted in Fig. 5.6) are calculated for both cases. The dotted curves represent the values obtained by traditional filtering, and the solid ones represent the values obtained by fuzzy filtering. In Fig. 5.5, the dotted curves are above the solid curves. This means that the AE RMS values obtained using the traditional filters are larger than those obtained with fuzzy filtering. The difference could be caused by different computation algorithm, where fuzzy filtering generates fuzzy rules directly from the input-output data acquired from sensor, without traditional user-defined high-pass and low-pass filters. The mean values obtained by the two methods are almost same as shown in Fig. 5.6.

The second step consists of expanding the type-1 fuzzy system to a type-2 system. Because the AE signals used are relatively uncontaminated, uncertainty in the AE signal is much smaller than the raw AE signal value. The spreading percentage for clusters is confined to the range [0.0%; 0.01%] with a step size of 0.0001%. The spreading percentage for the consequent parameters is considered as 2%. The information on uncertainty in the five AE signal sets is shown in Fig. 5.7 between the type-2 fuzzy output upper boundary  $\overline{AE}$  (dotted curve) and the lower boundary  $\underline{AE}$  (dashed curve). The overall identified AE signal value  $AE_{fuzzy}$  is shown as solid curve.



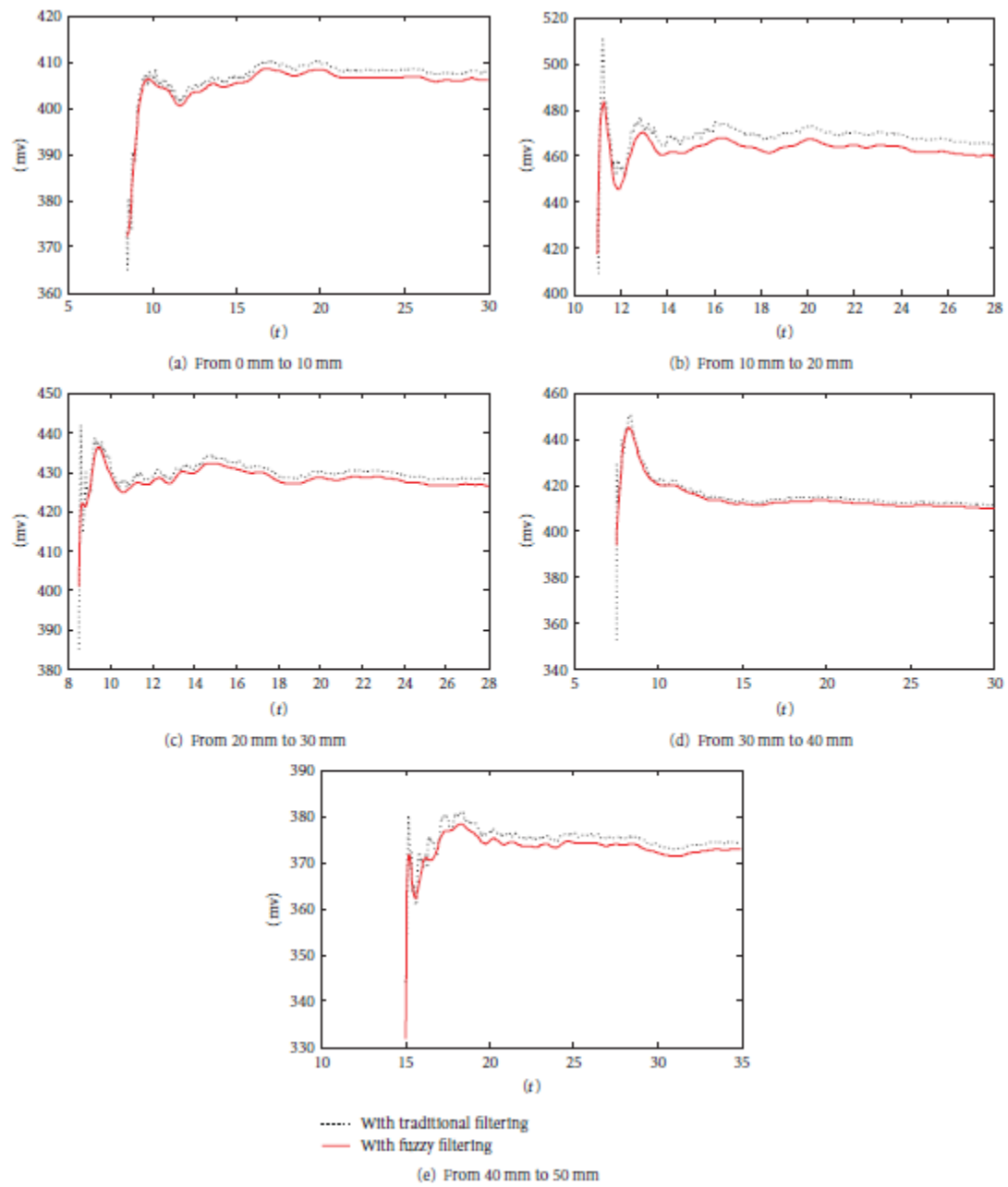


Fig. 5.5 AE RMS value for different cutting section

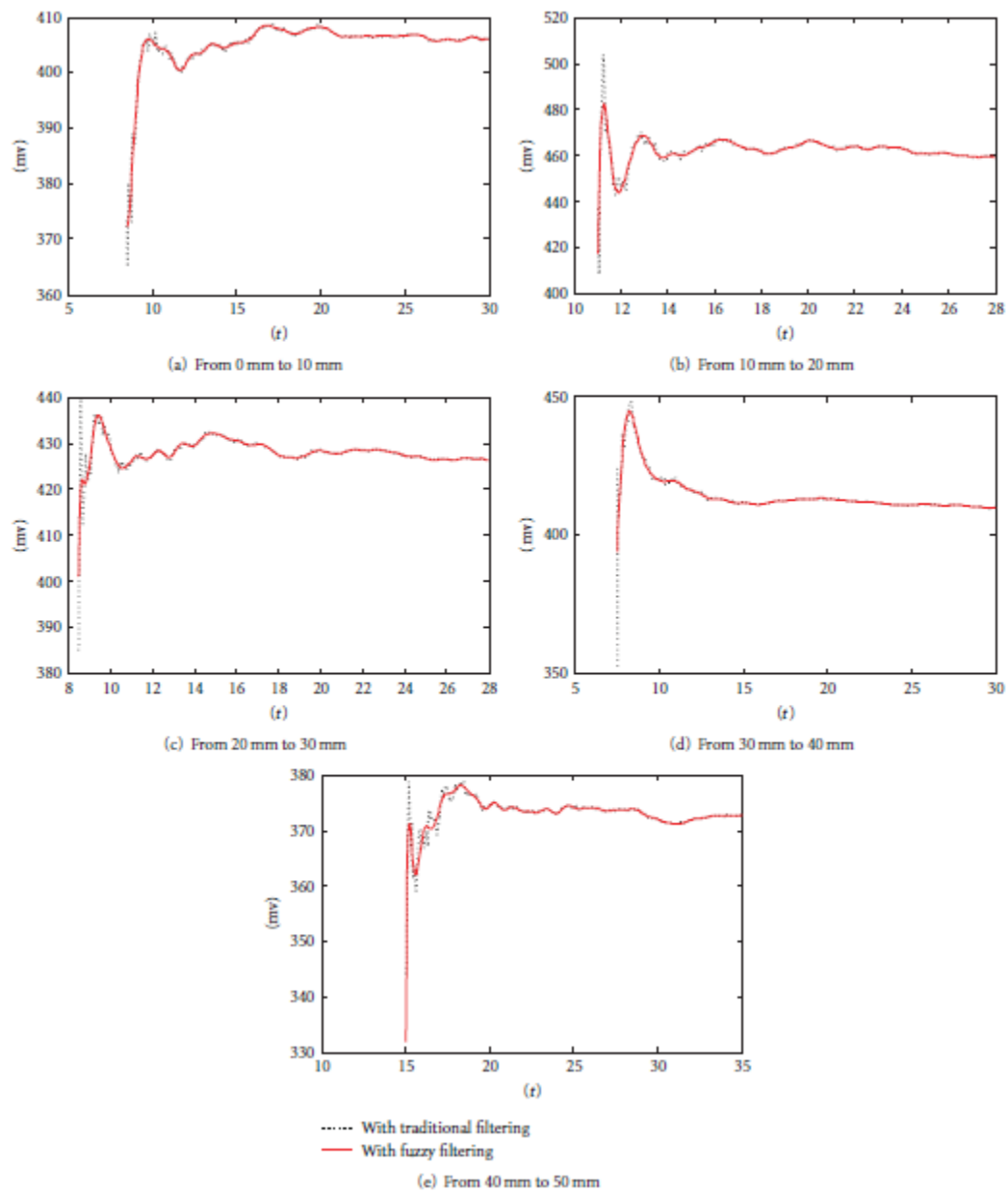


Fig. 5.6 AE mean value for different cutting section

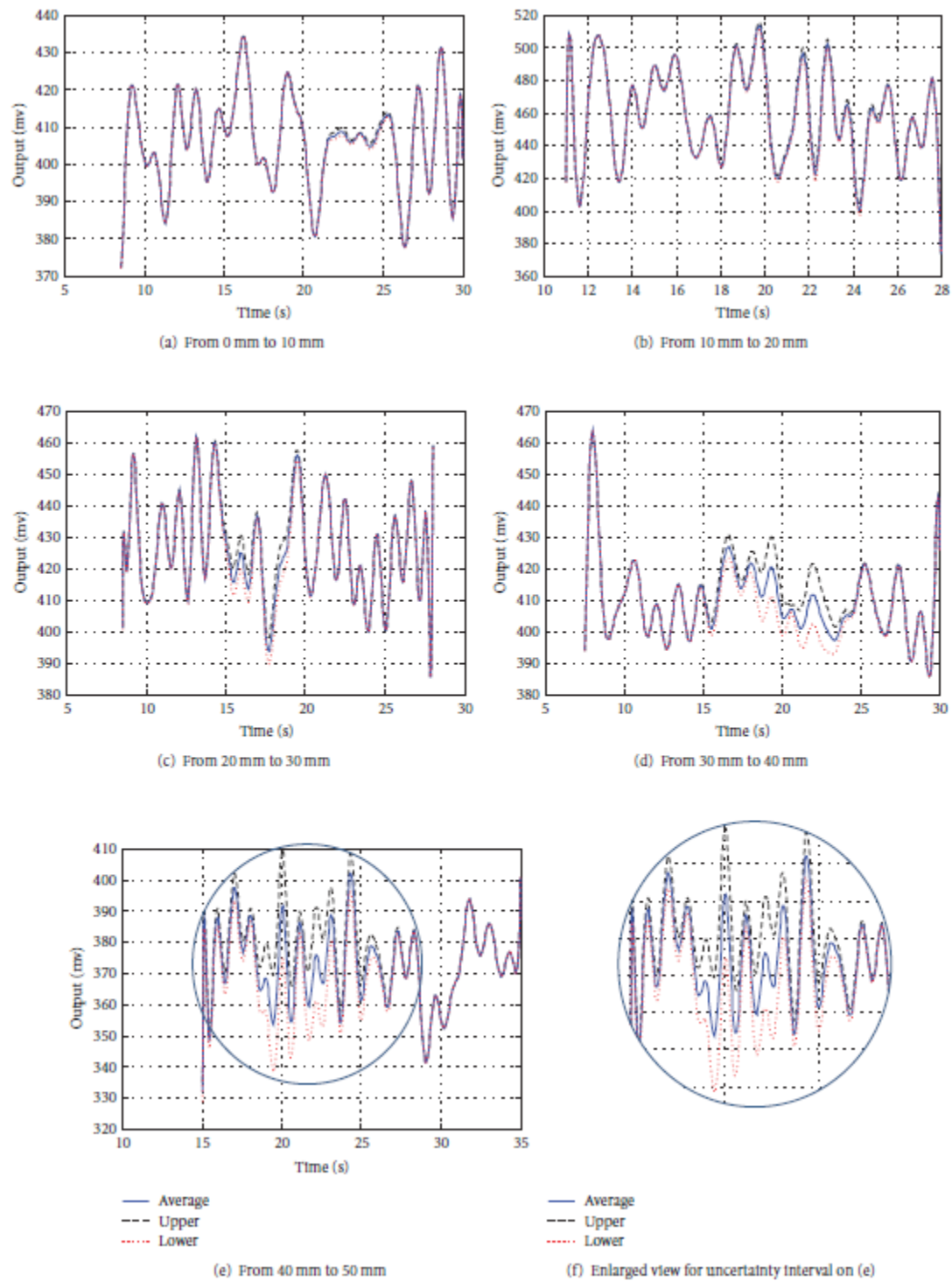


Fig. 5.7 Uncertainties in AE signal in different cutting sections

As indicated on Fig. 5.7, there is the uncertainty interval for each cutting section. The most significant is shown on Fig. 5.7(e). Table 5.2 summarizes the value of maximum and minimum variations between  $\overline{AE}$  and  $AE_{fuzzy}$ ,  $\underline{AE}$  and  $AE_{fuzzy}$ ,  $\overline{AE}$  and  $\underline{AE}$ , also  $AE_{fuzzy}$  and the raw AE signal value  $AE_{raw}$ , where, representatively,

$$V1 = |\overline{AE} - AE_{fuzzy}|, \quad (5.5)$$

$$V2 = |\underline{AE} - AE_{fuzzy}|, \quad (5.6)$$

$$V3 = |\overline{AE} - \underline{AE}| \quad (5.7)$$

and

$$V4 = |AE_{raw} - AE_{fuzzy}|. \quad (5.8)$$

The greatest variation of each cutting instant is V3 between the upper boundary and lower boundary of type-2 interval output. The biggest value is 24.7279 mv in the final cutting section, as same as that on Fig. 5.7(e). The last colon on Table 5.2 lists  $VB_b$  measured in the end of each cutting section.

As shown in Fig. 5.8 and 5.9, the development trends of maximum and minimum variations are the same as the tool wear trend. The maximum changes in tool wear condition and AE signal both occurred during the period when the cutting length was changed from 40 mm ~ 50 mm. It is observed that during the initial cutting period (cutting length from 0 ~ 40 mm), the variations of AE signal correspond to the initial stages of wear occurring. The period with the most significant variations (cutting section 40 ~ 50 mm) corresponds to the period of relatively rapid wear or failure of the cutting tool. Along with the increasing of uncertainty in AE signal, the development of wear is continuous and monotonically increasing. The sufficient information from AE uncertainty scheme can be used to make decision or investigate tool condition so as to enhance the reliability of tool wear estimation.

Table 5.2 The variations in modeling results from the five AE signal sets

Cutting section (mm)	Variation (mv)								$VB_B$ (mm)
	V1		V2		V3		V4		
	MAX	MIN	MAX	MIN	MAX	MIN	MAX	MIN	
0 ~10	1.1092	0.0262	1.0257	0.0003	2.1350	0.0108	13.6804	0.0670	0.050
10 ~20	4.7649	0.0214	4.5361	0.0006	9.3011	0.0018	28.7809	0.1597	0.100
20~ 30	5.9340	0.0604	5.660	0.0155	11.5940	0.0759	92.8960	0.1784	0.146
30~ 40	4.2204	0.0270	3.9954	0.0007	19.3474	0.0296	117.362	0.1980	0.196
40 ~ 50	12.630	0.0210	12.097	0.0001	24.7279	0.0040	128.294	0.0464	0.373

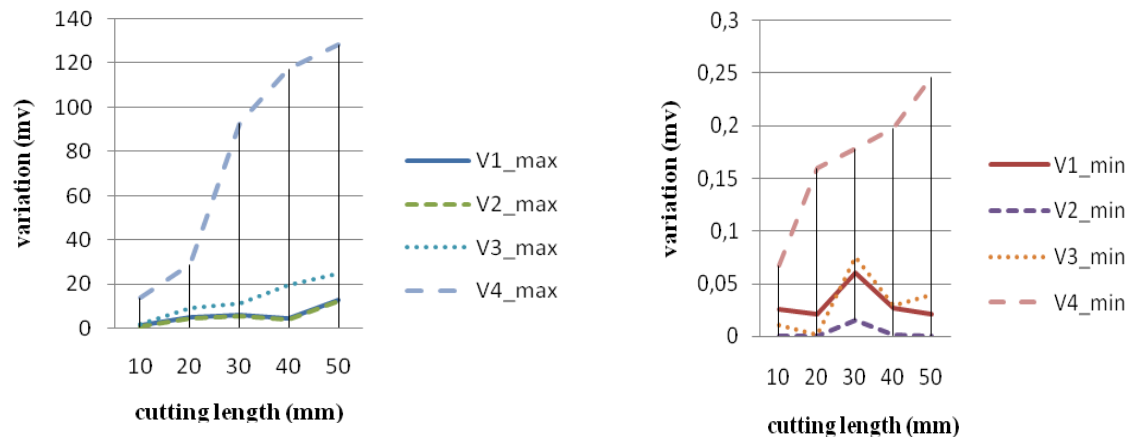


Fig. 5.8 Maximum and minimum variations in different cutting sections

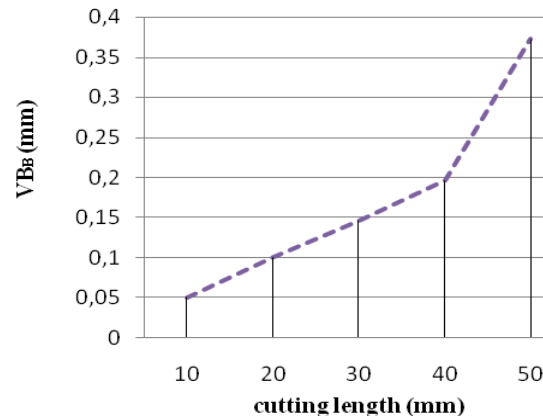


Fig. 5.9 Tool wears in different cutting sections

## 5.4 Conclusion

This paper presented a type-2 fuzzy modelling method to determine AE in precision machining. In the experimental study, AE from 5 cutting lengths were identified by using subtractive clustering based type-2 TSK identification algorithm. The interval outputs of type-2 fuzzy modelling provide the information of uncertainty in AE estimations. The sufficient information from AE uncertainty schemes correspond to the cutting tool states. This kind information is very useful to make decision or investigate tool condition so as to enhance the reliability of tool wear estimation. By applying type-2 fuzzy logic to AE based tool conditions monitoring, it is possible to automate on-line diagnosis of cutting tool condition.

Type-2 FLS can model and analyse the uncertainties in machining from the vague information obtained during machining process. The estimation of uncertainties can be used for proving the conformance with specifications for products or auto-controlling of machine system. The application of type-2 fuzzy logic on uncertainty estimation in high precision machining can enable the unmanned use of flexible manufacturing systems and machine tools. It has great meaning for continuous improvement in product quality, reliability and manufacturing efficiency in machining industry.

One limitation of this paper is the accuracy of the data sets. As mentioned, the data sets are discontinuing. To apply type-2 fuzzy logic to AE based tool conditions monitoring during the machining process, accurate and continue data is a necessary. Another limitation of the results obtained in this paper can be that the tool life determination is carried out solely using the AE information. Cutting force could be combined with AE to effectively determinate cutting tool life at a higher precision scale.

## Appendix A: Subtractive clustering

Subtractive clustering identification algorithm [41] is to estimate both the number and initial location of cluster centers and extract the TSK fuzzy rules from input/output data. Subtractive clustering operates by finding the optimal data point to define a cluster centre based on the density of surrounding data points. This method is a fast clustering method designed for high dimension problems with a moderate number of data points. This is because its computation

grows linearly with the data dimension and as the square of the number of data points. A brief description of Chiu's subtractive clustering method is as follows:

Consider a collection of  $q$  data points  $\{x_1, x_2, \dots, x_n\}$  specified by  $m$ -dimensional  $x_j$ . Without loss of generality, assume the feature space is normalized so that all data are bounded by a unit hypercube. Calculate potential for each point by using equation below:

$$p_i = \sum_{j=1}^q e^{-\alpha \|x_i - x_j\|^2}, \alpha = 4/r_a^2 \quad (5A-1)$$

where  $\|\cdot\|$  denotes the Euclidean distance. It is noteworthy that only the fuzzy neighbourhood within the radius  $r_a$  to the measure of potential.

After the potential of every data point has been computed, the data point with the highest potential is selected as the first cluster center. Assume  $x_1^*$  is the location of the first cluster center, and  $p_1^*$  is its potential value, then revise the potential of each data point  $x_j$  by the formula

$$p_i \Leftarrow p_i - p_1^* e^{-\beta \|x_i - x_1^*\|^2} \quad (5A-2)$$

where  $\beta = 4/r_b^2$  and  $r_b = \eta r_a$ .

When the potential of all data points have been reduce by (10), the data point with the highest remaining potential is selected as the second cluster center. Then further reduce the potential of each data points. Generally, after  $k$ th cluster center has been obtained, the potential of each data point is revised by formula

$$p_i \Leftarrow p_i - p_k^* e^{-\beta \|x_i - x_k^*\|^2} \quad (5A-3)$$

where  $x_k^*$  is the location of the  $k$ th cluster center and  $p_k^*$  is its potential value.

The process of acquiring new cluster center and revising potential repeats by using the following criteria:

if  $p_k^* > \varepsilon p_1^*$ , accept  $x_k^*$  as a cluster center and continue.

else if  $p_k^* < \varepsilon p_1^*$ , reject  $x_k^*$  and end the clustering process.

else let  $d_{\min}$  = shortest of the distances between  $x_k^*$  and all previously found cluster centers.

if  $\frac{d_{\min}}{r_a} + \frac{p_k^*}{p_1^*} \geq 1$  accept  $x_k^*$  as a cluster center and continue.

else reject  $x_k^*$  and set the potential at  $x_k^*$  to 0. Select the data point with the next highest potential as the new  $x_k^*$  and reset.

## Appendix B: Type-2 TSK fuzzy inference engine

For the most general structure of T2 TSK FLS – Model I, antecedents are T2 fuzzy sets and consequents are T1 fuzzy sets. Membership grades are interval sets, i.e.,

$$\mu_v^k = \left[ \mu_v^k, \bar{\mu}_v^k \right] \quad (5B-1)$$

where  $\mu_v^k$  and  $\bar{\mu}_v^k$  are the lower value and upper value of the  $v$ th input variable in the  $k$ th rule.

The explicit dependence of the total firing interval for  $k$ th rule can be computed as:

$$f^k = \mu_1^k(x_1) \cap \mu_2^k(x_2) \cap \dots \cap \mu_n^k(x_n) \quad (5B-2)$$

$$\bar{f}^k = \bar{\mu}_1^k(x_1) \cap \bar{\mu}_2^k(x_2) \cap \dots \cap \bar{\mu}_n^k(x_n) \quad (5B-3)$$

where variable  $f^k$  and  $\bar{f}^k$  denote lower value and upper value of fire strength. The symbol  $\cap$  is a conjunction operator, which is a T-norm. It can be either MIN operator  $\wedge$  or product operator  $*$ .

The interval value of the consequent of the  $k$ th rule  $w^k$  is  $w^k = [w_l^k, w_r^k]$ , where

$$w_l^k = \sum_{j=1}^n c_j^k x_j + c_0^k - \sum_{j=1}^n s_j^k |x_j| - s_0^k \quad (5B-4)$$

$$w_r^k = \sum_{j=1}^n c_j^k x_j + c_0^k + \sum_{j=1}^n s_j^k |x_j| - s_0^k \quad (5B-5)$$

Here  $w_l^k$  and  $w_r^k$  denote lower and upper values of consequent output for  $k$ th rule.  $c_j^k$  and  $s_j^k$  denotes the centre (mean) and the spread of fuzzy number  $\tilde{p}_j^k$ .



So, the extended output of the IT2 TSK FLS can be calculated by using following equation:

$$\begin{aligned} \tilde{w} &= [w_l, w_r] \\ &= \int_{w^1 \in [w_l^1, w_r^1]} \dots \int_{w^n \in [w_l^n, w_r^n]} \int_{f^1 \in [f_-^1, \bar{f}^1]} \dots \int_{f^M \in [f_-^M, \bar{f}^M]} \frac{1}{\frac{\sum_{k=1}^M f^k w^k}{\sum_{k=1}^M f^k}} \end{aligned} \quad (5B-6)$$

Hence  $\tilde{w} = [w_l, w_r]$  is an interval type-1 set, the two endpoints  $w_l$  and  $w_r$  can be obtained by use equations below:

$$w_l = \frac{\sum_{k=1}^n f_l^k w_l^k}{\sum_{k=1}^n f_l^k}, \quad w_r = \frac{\sum_{k=1}^n f_r^k w_r^k}{\sum_{k=1}^n f_r^k} \quad (5B-7)$$

This interval set of the output has the information about the uncertainties that are associated with the crisp output, and this information can only be obtained by working with T2 TSK FLS. To compute  $\tilde{w}$ , it's two end-points  $w_l$  and  $w_r$  must be computed.

In order to compute  $w_l$  and  $w_r$ ,  $f_l^k$  and  $f_r^k$  have to be determined.  $w_l$  and  $w_r$  can be obtained by using the iterative procedure *KM Algorithm* [30]. Here, the computation procedure for  $w_l$  and  $w_r$  is briefly provided as following:

Without loss of generality, assume that the pre-computed  $w_r^k$  are arranged in ascending order:  $w_r^1 \leq w_r^2 \leq \dots \leq w_r^m$ , then,

Step 1: Compute  $w_r$  in (5B-7) by initially setting

$$f_r^k = \left( f_-^k + \bar{f}^k \right) / 2 \quad (5B-8)$$

for  $k = 1, \dots, R$ , where  $f_-^k$  and  $\bar{f}^k$  have been previously computed using eq. (5B-2) and eq.(5B-3), respectively, and let  $w_r' \equiv w_r$ .

Step 2: Find  $y$  ( $1 \leq y \leq m-1$ ) such that  $w_r^y \leq w_r' \leq w_r^{y+1}$ .

Step 3: Compute  $w_r$  in with  $f_r^k = f_-^k$  for  $k \leq y$  and  $f_r^k = \bar{f}^k$  for  $k > y$ , and let  $w_r'' \equiv w_r$ .

Step 4: If  $w_r'' \neq w_r'$ , then go to Step 5. If  $w_r'' = w_r'$ , then stop. And set  $w_r'' \equiv w_r$ .

Step 5: Set  $w_r' = w_r''$ , and return to Step 2.

The procedure for computing  $w_l$  is very similar to the one just given for  $w_r$ . Replace  $w_r^k$  by  $w_l^k$ , and compute  $w_l$ . In Step 2 find  $z$  ( $1 \leq z \leq m-1$ ) such that  $w_l^z \leq w_l' \leq w_l^{z+1}$ . Additionally, in Step 3, compute  $w_l$  with  $f_r^k = f_l^k$  for  $k \leq z$  and  $f_r^k = \bar{f}^k$  for  $k > z$ .

In an interval type-2 TSK FLS, output  $\tilde{w}$  is an interval type-1 fuzzy set, so the crisp output of any interval type-2 TSK FLS can be obtained by using the average value of  $w_l$  and  $w_r$ , i.e.,

$$w^* = \frac{w_l + w_r}{2} \quad (5B-9)$$

## Reference

- [1] L. H. Wang, Robert X. Gao, “*Condition Monitoring and Control for Intelligent Manufacturing*”, Springer Series in Advanced Manufacturing, ISBN: 978-1-84628-268-3, 2006.
- [2] K. Iwata, T. Moriwaki, “Application of acoustic emission measurement to in-process sensing of tool wear”, *Ann. CIRP*, vol. 26, no. 1-2, pp. 19–23, 1977.
- [3] S. Liang, D. Dornfeld, “Tool wear detection using time series analysis of acoustic emission”, *J. Eng. Ind. Trans. ASME*, vol. 111, no. 3, pp. 199-205, 1989.
- [4] H. Ravindra, Y. Srinivasa and R. Krishnamurthy, “Acoustic emission for tool condition monitoring in metal cutting”, *Wear*, vol.212, no. 1, pp. 78–84, 1997.
- [5] E. Emel and E. Kannatey-Asibu, Jr., “Tool failure monitoring in turning by pattern recognition analysis of AE signal”. *Trans. ASME J. Eng. Ind.* vol. 110 pp. 137–145, 1988.
- [6] X. Li, Z. Yuan, “Tool wear monitoring with wavelet packet transform-fuzzy clustering method”, *Wear*, vol. 219, no. 2, pp.145–154, 1998.
- [7] A. Menon, Z. Boutaghou, “Time-frequency analysis of tribological systems—Part I: implementation and interpretation”, *Tribol. Int.* vol. 31 no. 9, pp. 501–510, 1998.
- [8] C. Heil and D. Walnut, “Continuous and discrete wavelet transforms”, *SIAM Rev.*, vol. 31, no. 4, pp. 628–666, 1989.
- [9] E. Serrano and M. Fabio, "Application of the wavelet transform to acoustic emission signals

- processing”, *IEEE Trans. Signal Proces*, vol.44, no. 5, pp. 1270–1275, 1996.
- [10] S. Kamarthi, S. Kumara, P. Cohen, “Wavelet representation of acoustic emission in turning process”, *Intell. Eng. Syst. Artif. Neural Netw.* Vol. 5 pp. 861–866, 1995.
- [11] X. Li, “Real-time detection of the breakage of small diameter drills with wavelet transform”, *Int. J. Adv. Manuf. Technol.*, vol. 14, no. 8, pp. 539–543, 1998.
- [12] I. Nur Tansel et al., "Detection of tool failure in end milling with wavelet transformations and neural networks (WT-NN) ", *Int. J. Mach. Tools Manufact.*, vol. 35, no.4, pp. 1137–1147, 1995.
- [13] N. Kasashima et al., "Online failure detection in face milling using discrete wavelet transform”, *Ann. CIRP*, vol. 44, no. 1, pp. 483–487, 1995.
- [14] X. Li, "A brief review: acoustic emission method for tool wear monitoring during turning”, *International Journal of Machine Tools and Manufacture*, vol. 42, no.2, pp. 157-165, 2002.
- [15] S. Darenfed and S. Wu, "Polynomial learning networks for cutting tool diagnosis in machining operations”, *Trans. Can. Soc. Mech. Eng.*, vol.16, no. 2, pp. 147–163, 1992.
- [16] Y. Quan, M. Zhou and Z. Luo, "On-line robust identification of tool-wear via multi-sensor neural-network fusion", *Engineering Applications of Artificial Intelligence*, vol. 11, no.6, pp. 717-722, 1998.
- [17] Q. Ren, L. Baron and M. Balazinski, "Fuzzy Identification of Cutting Acoustic Emission with Extended Subtractive Cluster Analysis", *Nonlinear Dynamics*, DOI: 10.1007/s11071-011-0173-5, 2011.
- [18] X. Li, Y. Yao and Z. Yuan, "On-line tool condition monitoring system with wavelet fuzzy neural network", *Journal of Intelligent Manufacturing*, vol. 8, pp.271-276, 1997.
- [19] R. J. Kuo and P. H. Cohen, "Intelligent tool wear estimation system through artific neural networks and fuzzy modeling", *Artificial Intelligence in Engineering*, vol. 12, no.3, pp. 229-242, 1998
- [20] P. Lezanski, "An intelligent system for grinding wheel condition monitoring", *Journal of Materials Processing Technology*, vol.109, no.3, pp. 258-263, 2001.
- [21] J. Sun, M. Rahman, Y. S. Wong and G. S. Hong, "Multiclassification of tool wear with support vector machine by manufacturing loss consideration", *International Journal of Machine Tools and Manufacture*, vol. 44, no. 11, pp. 1179-1187, 2004.

- [22]I. Deiab, K. Assaleh and F. Hammad, "On modeling of tool wear using sensor fusion and polynomial classifiers", *Mechanical Systems and Signal Processing*, vol.23, no. 5, pp. 1719-1729, 2009.
- [23]L. A. Zadeh, "Fuzzy Sets", *Information and Control*, vol. 8, pp.338-353, 1965
- [24]T. Takagi and M. Sugeno, "Fuzzy Identification of Systems and Its Applications to Modeling and Control", *IEEE Transaction on Systems, Man, and Cybernetics*, vol. 15, no. 1, pp. 116-132, 1985.
- [25]M. Sugeno and G. Kang, "Structure Identification of Fuzzy Model", *Fuzzy Sets and Systems*, vol. 28, no. 1, pp. 15-33, 1988.
- [26]T. Johansen, B Foss, "Identification of Non-Linear System Structure and Parameters Using Regime Decomposition", *Automatic*, vol. 31, no.2, pp. 321–326, 1995.
- [27]D. Füssel, P Ballé, R. Isermann, Closed Loop Fault Diagnosis Based on A Nonlinear Process Model and Automatic Fuzzy Rule Generation, *4th IFAC Sym. on Fault Det., Sup. and Safety for Tech. Proc.*, 1997.
- [28]L. A. Zadeh, "The Conception of A Linguistic Variable and Its Application in Approximate Reasoning – I", *Information Science*, vol. 8, pp.199-249, 1975.
- [29]M. Mizumoto, M. and K. Tanaka, "Fuzzy sets of type 2 under algebraic product and algebraic sum", *Fuzzy Sets and Systems*, vol. 5, no. 3, pp. 277-90, 1981.
- [30]N. N. Karnik, and J. M. Mendel, "Introduction to Type-2 Fuzzy Logic Systems", Technical report, University of Southern California, <http://sipi.usc.edu/-Mendel/report>, 1998.
- [31]R. I. John, "Type-2 Fuzzy Sets: An Appraisal of Theory and Applications", *International Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, vol. 6, no. 6, pp. 563-576, 1998.
- [32]L. A. Zadeh, "Fuzzy Logic = Computing with Words", *IEEE transactions on fuzzy systems*, vol.4, no. 2, pp. 103-111, 1996.
- [33]L. A. Zadeh, "From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions", *IEEE transactions in circuits and systems*, vol. 45, no.1, pp. 105-119, 1999.
- [34]J. M. Mendel, "Uncertain Rule-Based Fuzzy logic Systems – Introduction on New Directions", prentice hall PTR, upper saddle river, NJ, 2001.
- [35]H. Hagra, "Type-2 FLCs: a new generation of fuzzy controllers", *IEEE Computational*

- Intelligence Magazine*, vol. 2, no. 1, pp. 30-43, 2007.
- [36] D. Wu and W. W. Tan, "A simplified type-2 fuzzy logic controller for real-time control", *ISA Transactions*, vol. 45, no. 4, pp. 503-16, 2006.
- [37] Q. Liang, and J. M. Mendel, "An Introduction to Type-2 TSK Fuzzy Logic Systems," *Proc. IEEE FUZZ Conference*, Seoul, Korea, 1999.
- [38] Mendel J. M., Hagrass H., John R. I., "Standard background material about interval type-2 fuzzy logic systems that can be used by all authors", <http://ieee-cis.org/files/standards.t2.win.pdf>.
- [39] J. M. Mendel, "Advances in type-2 fuzzy sets and systems", *Information Sciences*, vol.177, pp. 84–110, 2007.
- [40] Q. Ren, L. Baron and M. Balazinski, "Type-2 Takagi-Sugeno-Kang Fuzzy Logic Modeling using subtractive clustering", *The 25th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2006)*, Montreal, Canada, pp.120-125, 2006.
- [41] Chiu S. L. "Fuzzy Model Identification Based on Cluster Estimation. J. on Int. Fuzzy Sys. vol. 2, 267–278. 1994.
- [42] Q. Ren, L. Baron, M. Balazinski and K. Jemelniak, "*Tool Condition Monitoring Using the TSK Fuzzy Approach Based on Subtractive Clustering Method*", *News Frontiers in Applied Artificial Intelligence*, pp. 52-61, Springer Berlin, 2008.
- [43] Q. Ren, L. Baron, M. Balazinski and K. Jemelniak, "TSK fuzzy modeling for tool wear condition in turning processes: An experimental study", *Engineering Applications of Artificial Intelligence*, vol. 24, no. 2, pp. 260-265, 2011.
- [44] Q. Ren, L. Baron, K. Jemelniak and M. Balazinski, "Acoustic Emission Signal Feature Analysis Using Type-2 Fuzzy logic System", *The 29th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2010)*, Toronto, Canada, July 12 - 14, pp.1-6, 2010
- [45] Q. Ren, L. Baron and M. Balazinski, "Uncertainty Prediction for Tool Wear Condition Using Type-2 TSK Fuzzy Approach", *IEEE International Conference on Systems, Man, and Cybernetics (SMC 2009)*, San Antonio, Texas, USA, pp. 660-665, 2009.
- [46] Q. Ren, L. Baron and M. Balazinski, "Application of Type-2 Fuzzy Estimation on Uncertainty in Machining: An Approach on Acoustic Emission during Turning Process", *The 28th North American Fuzzy Information Processing Society Annual Conference (NAFIPS*

- 2009), Cincinnati, Ohio, USA, 2009.
- [47] Q. Ren, L. Baron, K. Jemielniak and M. Balazinski, "Modelling of Dynamic Micromilling Cutting Forces Using Type-2 Fuzzy Rule-Based System", *2010 IEEE World Congress on Computational Intelligence International Conference on Fuzzy Systems (WCCI 2010)*, Barcelona, Spain, pp. 2311-2317, 2010.
- [48] Q. Ren, M. Balazinski and L. Baron, "Type-2 TSK Fuzzy Logic System and Its Type-1 Counterpart", *International journal of computer applications*, vol.20, no. 6, pp. 8-13, 2011.
- [49] K. Demirli, S. X. Cheng and P. Muthukumaran, "Subtractive clustering based on modelling of job sequencing with parametric search", *Fuzzy Sets and Systems*, vol.137, pp. 235-270, 2003
- [50] Q. Ren, L. Baron and M. Balazinski, "Type-2 Fuzzy Uncertainty Estimation on Acoustic Emission for Tool Wear Assessment", submitted for publication, 2011

## CHAPTER 6      HIGH-ORDER INTERVAL TYPE-2 TAKAGI-SUGENO-KANG FUZZY LOGIC SYSTEM AND ITS APPLICATION IN ACOUSTIC EMISSION SIGNAL MODELING IN TURNING PROCESS

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**Abstract** – Type-2 fuzzy logic systems (FLSs) are gaining in popularity because of their capacity to handle rule uncertainties in a more complete way. Moreover, higher order interval type-2 (IT2) FLS can reduce drastically the number of rules needed to perform the approximation and improve transparency and interpretation in many high dimensional systems. This paper presents architecture and inference engine of generalized IT2 Takagi-Sugeno-Kang (TSK) FLS, and the design method of higher order IT2 FLS. An experimental acoustic emission (AE) signal modelling using a second order IT2 TSK FLS in a turning process is used to demonstrate the differences between the first order and second order IT2 FLSs, and the advantage and efficiency of high order IT2 FLS. Estimation of uncertainty of AE could be of great value to a decision maker and be used to monitor tool wear condition during machining process.

**Keyword** – *Type-2 fuzzy logic system, high order, fuzzy modelling, acoustic emission*

### 6.1 Introduction

Based on Zadeh's conception of type-2 (T2) fuzzy sets and extension principle [1, 2], practical algorithms for conjunction, disjunction and complementation operations of T2 fuzzy sets are obtained by extending previous studies [3-6]. A general formula was developed for the extended

composition of T2 relations which is considered as an extension of the Type-1 (T1) composition [7, 8]. Based on this formula, a complete T2 fuzzy logic theory with the handling of uncertainties was established [9, 10]. By using the discrete probability theory, embedded interval valued T2 fuzzy sets was introduced and it made T2 fuzzy sets easy to understand and explain [11, 12]. First order interval type-2 (IT2) Takagi-Sugeno- Kang (TSK) FLS and its structures was presented in 1999 [13]. Because the universal approximation property and the capability of handling rule uncertainties in a more complete way, IT2 FLSs are gaining in popularity. Recent Industrial applications of type-2 fuzzy sets and systems can be found in [14].

Zero order and first order IT2 TSK FLSs, as its T1 counterpart [15], suffer the curse of dimensionality – the number of rules increases exponentially with the number of input variables and the number of MFs per variable. Higher order consequent TSK rules can reduce drastically the number of rules needed to perform the approximation, and improve transparency and interpretation in many high dimensional systems. High order IT2 FLS was proposed by authors to solve this kind problem [16]. Higher order IT2 TSK FLSs can not only handle uncertainties within the fuzzy system as first order IT2 FLSs, but also reduce the number of rules needed to develop robust and reliable solutions for the problems.

This paper presents architecture and inference engine of generalized IT2 TSK FLS, and design method of higher order IT2 FLS. An experimental acoustic emission signal modelling using a second order IT2 TSK FLS in turning process is given to demonstrate the advantage and efficiency of high order IT2 FLS. Estimation of uncertainty of AE could be of great value to a decision maker and be used to investigate tool wear condition during machining process.

The structure of this paper includes 6 sections. Section 6.2 is reviews of recent TSK FLSs, both T1 and IT2 TSK FLS. The generalized IT2 TSK FLS and its inference engine are in Section 6.3. In Section 6.4, a design method for high order IT2 TSK FLS is presented in detail. An example of application of a second order IT2 TSK FLS on uncertainty prediction of acoustic emission (AE) in high precision machining is given in Section 6.5. A comparison of results with that of first order IT2 system is given to demonstrate advantage and efficiency of high order IT2 FLS. Conclusions are in Section 6.6.



Because a multi-input multi-output (MIMO) fuzzy system can be viewed as a set of multi-input single-output (MISO) fuzzy system, without loss of generality, fuzzy systems in this paper are only MISO mappings.

## 6.2 Theoretical Foundations

### 6.2.1 Type-1 TSK FLS

A T1 TSK model is described by fuzzy IF-THEN rules which represent input-output relations of a system. For a T1 TSK model, its  $k$ th rule can be expressed as:

IF  $x_1$  is  $Q_1^k$  and  $x_2$  is  $Q_2^k$  and ... and  $x_n$  is  $Q_n^k$ ,

THEN  $Z$  is  $w^k = f_k(\vec{x})$ ,

where  $x_1, x_2, \dots, x_n$  and  $Z$  are linguistic variables;  $\vec{x} = [x_1, x_2, \dots, x_n]$ ,  $Q_1^k, Q_2^k, \dots, Q_n^k$  are the fuzzy sets on universe of discourses,  $w^k$  is a crisp output of the  $k$ th rule and  $w^k = f_k(\vec{x})$  is a constant or a polynomial function of input variables, while  $n$  is the total number of variable.

The generalized  $k$ th rule of  $m$ -order T1 TSK model also can be described by the following expression [17]:

IF  $x_1$  is  $Q_1^k$  and  $x_2$  is  $Q_2^k$  and ... and  $x_n$  is  $Q_n^k$ ,

THEN  $Z$  is  $w^k = \sum_{j=0}^m \sum_{i=0}^m \dots \sum_{q=0}^m p_{ji\dots q}^k x_1^j x_2^i \dots x_n^q$

where  $p_{ji}^k$  represents  $i$ -order coefficients for  $j = 0, 2, \dots, m, i = 0, 2, \dots, m, \dots, q = 0, 2, \dots, m$  in the  $k$ th rule where  $j, i, \dots, q$  are the order of variables.

Gaussian functions are usually chosen as MFs. A T1 Gaussian MF can express by using formula for the  $v$ th variable:

$$Q_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*}}{\sigma} \right)^2 \right], \quad (6.1)$$

where  $x_v^{k*}$  is mean of the  $v$ th input feature in the  $k$ th rule for  $v \in [0, n]$ ,  $\sigma$  is the standard deviation of Gaussian MF.

### 6.2.2 First-order IT2 Type-2 TSK FLS

A  $k$ th rule in a first-order IT2 TSK fuzzy MISO system is expressed as

$$\begin{aligned} \text{IF } x_1 \text{ is } \tilde{Q}_1^k \text{ and } x_2 \text{ is } \tilde{Q}_2^k \text{ and } \dots \text{ and } x_n \text{ is } \tilde{Q}_n^k, \\ \text{THEN } Z \text{ is } \tilde{w}^k = p_0 + p_1 x_1 + p_2 x_2 + \dots + p_n x_n \end{aligned}$$

where  $\tilde{Q}_1^k, \tilde{Q}_2^k, \dots, \tilde{Q}_n^k$  are IT2 fuzzy sets on universe of discourses, fuzzy numbers  $\tilde{p}_0, \tilde{p}_1, \dots, \tilde{p}_n$  are consequent parameters, and  $\tilde{w}^k$  output from the  $k$ th IF-THEN rule in an FLS with a total of  $R$  rules.

### 6.3 Generalized IT2 TSK fuzzy system

In a generalized IT2 TSK FLS, the antecedent or consequent membership functions (MFs) are IT2 fuzzy sets and its consequent part is a mathematical function that can be a constant or any order polynomial function of input variables. The  $k$ th rule of  $m$ -order IT2 TSK FLS can be expressed as

$$\begin{aligned} \text{IF } x_1 \text{ is } \tilde{Q}_1^k \text{ and } x_2 \text{ is } \tilde{Q}_2^k \text{ and } \dots \text{ and } x_n \text{ is } \tilde{Q}_n^k, \\ \text{THEN } Z \text{ is } w^k = \sum_{j=0}^m \sum_{i=0}^m \dots \sum_{q=0}^m \tilde{p}_{ji\dots q}^k x_1^j x_2^i \dots x_n^q \end{aligned}$$

where  $\tilde{p}_{ji\dots q}^k$  represents consequent parameters which are fuzzy numbers.

As Buckley noted [18], a  $m$ -order polynomial consequent of  $k$ th TSK fuzzy rule can be generalized as follows:

$$w^k = \tilde{p}_0 + \tilde{p}_1 \overset{\rightarrow}{x} + \overset{\rightarrow}{x} \tilde{P}_2 \overset{\rightarrow}{x} + \dots + \left\langle \tilde{P}_m^* (\overset{\rightarrow}{x} \otimes \dots \otimes \overset{\rightarrow}{x}) \right\rangle, \quad (6.2)$$

where  $\tilde{p}_0^k$  is a zero-order coefficient,  $\tilde{p}_1^k$  is a vector of first order coefficients,  $\tilde{p}_2^k$  is a triangular matrix of second order coefficients and  $\tilde{p}_m^k$  is a triangular  $m$ -dimensional matrix of coefficients. These coefficients are fuzzy numbers.

A type-2 Gaussian MF can express by using formula for the  $v$ th variable:

$$\tilde{Q}_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^{k*} (1 \pm a_v^k)}{\sigma^k} \right)^2 \right], \quad (6.3)$$

where  $a_v^k$  is spread percentage of mean  $x_v^{k*}$  as shown in Fig. 1 [19].  $\sigma^k$  is the standard deviation of Gaussian MF of the  $k$ th rule.

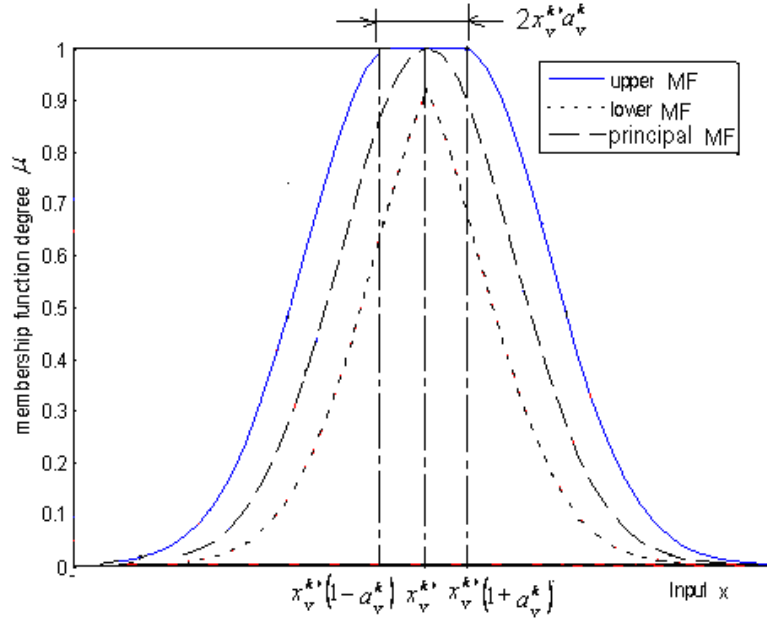


Fig. 6.1 Spread of mean of the  $v$ th variable in the  $k$ th rule

From the structure of the generalized IT2 TSK FLS, when  $m=0$ ,  $w^k$  is a constant, the system is a zero order IT2 TSK FLS. When  $m=1$ ,  $w^k$  is a first order polynomial function, the system is a first order IT2 TSK FLS. When  $m \geq 2$ ,  $w^k$  is a more than second order polynomial function, the system is a higher order IT2 TSK FLS [16]. Higher order and the first order IT2 TSK FLS have

the same antecedent. The difference is the order in consequent equation. Moreover, the computation of interval values of the consequent output for each rule is more complicated in a high order system. A larger reduction in number of rules is possible with higher order system identification technique [17].

Mendel in his book [12] proposes a complete computation method for first order IT2 TSK FLS. It can be generalized to  $m$ -order T2 TSK FLS inference computation.

For the most general structure of T2 TSK FLS – Model I, antecedents are T2 fuzzy sets and consequents are T1 fuzzy sets. Membership grades are interval sets, i.e.,

$$\mu_v^k = \left[ \mu_v^k, \mu_v^k \right] \quad (6.4)$$

where  $\mu_v^k$  and  $\mu_v^k$  are the lower value and upper value of the  $v$ th input variable in the  $k$ th rule.

The explicit dependence of the total firing interval for  $k$ th rule can be computed as:

$$f^k = \mu_1^k(x_1) \cap \mu_2^k(x_2) \cap \dots \cap \mu_n^k(x_n) \quad (6.5)$$

$$\bar{f}^k = \bar{\mu}_1^k(x_1) \cap \bar{\mu}_2^k(x_2) \cap \dots \cap \bar{\mu}_n^k(x_n) \quad (6.6)$$

where variable  $f^k$  and  $\bar{f}^k$  denote lower value and upper value of fire strength. The symbol  $\cap$  is a conjunction operator, which is a T-norm. It can be either MIN operator  $\wedge$  or product operator  $*$ .

The interval value of the consequent of the  $k$ th rule  $w^k$  is  $w^k = [w_l^k, w_r^k]$ , where

$$w_l^k = \sum_{j=0}^m \sum_{i=0}^m \dots \sum_{q=0}^m c_{ji\dots q}^k x_1^j x_2^i \dots x_n^q - \sum_{j=0}^m \sum_{i=0}^m \dots \sum_{q=0}^m s_{ji\dots q}^k | x_1^j x_2^i \dots x_n^q | \quad (6.7)$$

$$w_r^k = \sum_{j=0}^m \sum_{i=0}^m \dots \sum_{q=0}^m c_{ji\dots q}^k x_1^j x_2^i \dots x_n^q + \sum_{j=0}^m \sum_{i=0}^m \dots \sum_{q=0}^m s_{ji\dots q}^k | x_1^j x_2^i \dots x_n^q |. \quad (6.8)$$

Here  $w_l^k$  and  $w_r^k$  denote lower and upper values of consequent output for  $k$ th rule.  $c_{ji\dots q}^k$  and  $s_{ji\dots q}^k$  denotes mean and the spread of fuzzy number  $\tilde{p}_{ji\dots q}^k$ .

So, the extended output of the IT2 TSK FLS can be calculated by using following equation:

$$\begin{aligned} \tilde{w} &= [w_l, w_r] \\ &= \int_{w^1 \in [w_l^1, w_r^1]} \cdots \int_{w^n \in [w_l^n, w_r^n]} \int_{f^1 \in \begin{bmatrix} f^1 & \bar{f}^1 \\ - & \end{bmatrix}} \cdots \int_{f^M \in \begin{bmatrix} f^M & \bar{f}^M \\ - & \end{bmatrix}} \frac{1}{\frac{\sum_{k=1}^M f^k w^k}{\sum_{k=1}^M f^k}}. \end{aligned} \quad (6.9)$$

Hence  $\tilde{w} = [w_l, w_r]$  is an interval type-1 set, the two endpoints  $w_l$  and  $w_r$  can be obtained by use equations below:

$$w_r = \frac{\sum_{k=1}^n f_r^k w_r^k}{\sum_{k=1}^n f_r^k}, \quad w_l = \frac{\sum_{k=1}^n f_l^k w_l^k}{\sum_{k=1}^n f_l^k} \quad (6.10)$$

This interval set of the output has the information about the uncertainties that are associated with the crisp output, and this information can only be obtained by working with T2 TSK FLS. To compute  $\tilde{w}$ , its two end-points  $w_l$  and  $w_r$  must be computed.

In order to compute  $w_l$  and  $w_r$ ,  $f_l^k$  and  $f_r^k$  have to be determined.  $w_l$  and  $w_r$  can be obtained by using *KM Algorithm* -- the iterative procedure proposed by Karnik and Mendel [7]. Here, the computation procedure for  $w_l$  and  $w_r$  is briefly provided as following:

Without loss of generality, assume that the pre-computed  $w_r^k$  are arranged in ascending order:  $w_r^1 \leq w_r^2 \leq \dots \leq w_r^m$ , then,

Step 1: Compute  $w_r$  in (6.10) by initially setting

$$f_r^k = \left( \begin{matrix} f^k + \bar{f}^k \\ - \end{matrix} \right) / 2 \quad (6.11)$$

for  $k = 1, \dots, R$ , where  $f^k$  and  $\bar{f}^k$  have been previously computed using eq.(6.5) and eq.(6.6), respectively, and let  $w_r' \equiv w_r$ .

Step 2: Find  $y$  ( $1 \leq y \leq m-1$ ) such that  $w_r^y \leq w_r' \leq w_r^{y+1}$ .

Step 3: Compute  $w_r$  in (10) with  $f_r^k = f^k$  for  $k \leq y$  and  $f_r^k = \bar{f}^k$  for  $k > y$ , and let  $w_r'' \equiv w_r$ .

Step 4: If, the  $w_r'' \neq w_r'$  go to Step 5. If  $w_r'' = w_r'$ , then stop. And set  $w_r'' \equiv w_r'$ .

Step 5: Set  $w_r' = w_r''$ , and return to Step 2.

The procedure for computing  $w_l$  is very similar to the one just given for  $w_r$ . Replace  $w_r^k$  by  $w_l^k$ , and compute  $w_l$  in eq.(10). In Step 2 find  $z$  ( $1 \leq z \leq m-1$ ) such that  $w_l^z \leq w_l' \leq w_l^{z+1}$ . Additionally, in Step 3, compute  $w_l$  in eq.(10) with  $f_r^k = f_r^k$  for  $k \leq z$  and  $f_r^k = \bar{f}^k$  for  $k > z$ .

In an interval type-2 TSK FLS, output  $\tilde{w}$  is an interval type-1 fuzzy set, so the crisp output of any interval type-2 TSK FLS can be obtained by using the average value of  $w_l, w_r$ , i.e.,

$$w^* = \frac{w_l + w_r}{2} \quad (6.12)$$

The performance of a IT2 TSK FLS is evaluated using the following root-mean-square-error (RMSE):

$$RMSE = \sqrt{\frac{1}{g} \sum_{s=1}^g (W_{ss} - W_{sm})^2} \quad (6.13)$$

Here the initial system has a group of data with  $n$  vector,  $W_{ss}$  and  $W_{sm}$  are the system output and model output for  $s$ th vector,  $s \in [1, g]$  where  $g$  is the total number of input vector. The best model has the least RMSE.

## 6.4 Design method for high order IT2 TSK FLS

The higher order type-1 fuzzy system identification method was first proposed in [17]. Subtractive clustering based T1 TSK modelling [20, 21] involves generation of clusters using input/output data set for pre-initialized clustering parameters such as squash factor, cluster radius, accept ratio and reject ratio, and estimation of regression coefficients by least-square estimation. High order IT2 TSK FLS can be obtained directly from its T1 counterpart by considering the T1 MFs as principal MFs and assigning uncertainty to cluster centers and consequence parameters. The diagram of the algorithm based on subtractive clustering is shown in Fig. 6.2 which is same

as that of first-order IT2 TSK FLS presented in [19]. Type-1 Gaussian MFs is used as principal MFs to expand T1 TSK model to T2 TSK model. The proposed T2 TSK modelling identification algorithm is as following steps:

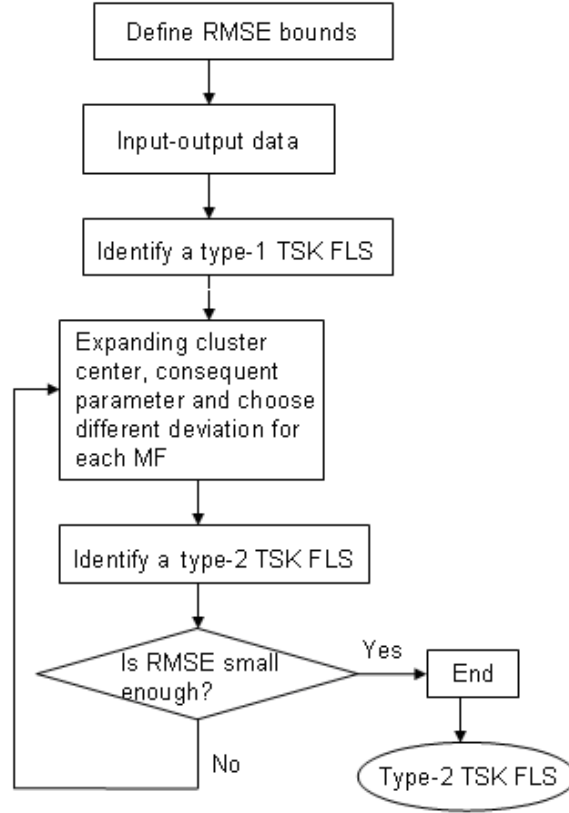


Fig. 6.2 Diagram of type-2 TSK FLS

Step 1: Use higher order type-1 fuzzy system identification method [16] using subtractive clustering to pre-identify a type-1 fuzzy model from input/output data.

Step 2: Use type-1 Gaussian MFs as principal MFs to expand T1 TSK model to T2 TSK model:

- Spread cluster centre to expand premise MFs from T1 fuzzy sets to T2 fuzzy sets using eq. (6.3) where  $a_v^k$  is spread percentage of cluster centre  $x_v^{k*}$  as depicted in Fig. 6.1.
- Spread the parameters of consequent to expand consequent parameters from certain value to fuzzy numbers:

$$p_{ji\dots q}^{\sim k} = p_{ji\dots q}^k (1 \pm b_{ji\dots q}^k), \quad (6.14)$$

where  $c_{ji\dots q}^k = p_{ji\dots q}^k$  and  $s_{ji\dots q}^k = p_{ji\dots q}^k b_{ji\dots q}^k$  in eq.(6.6) and eq.(6.7), where  $b_{ji\dots q}^k$  is the spread percentage of fuzzy numbers  $p_{ji\dots q}^{\sim k}$ .

Step 3: By using high order type-2 TSK FLS computation method proposed, calculate the interval value of the consequent for each input and obtain the two end-points of output interval set and average value of output.

Step 4: Calculate RMSE of this type-2 model. If RMSE is bigger than expected error limitation, go back to Step3 until satisfied.

## 6.5 Experimental study

The experiment of this paper was taken on the machine BOEHRINGER CNC Lathe [22]. The workpiece material is 10 Vol.% (TiB+TiC)/Ti-6Al-4V Metal Matrix Composites. TiC/Ti-6Al-4V where the microstructural response of cast Ti-6Al-4V based composite contains 10-12 Vol.% TiC reinforcement. This kind material is usually used on aerospace and military instruments for its high hardness, light weight, high bending strength, fracture toughness, higher modulus, and elevated temperature resistance and high wear resistance.

The cutting tool insert was carbide from SECO tools (CNMG 120408 MF1 CP200). Turning test was done on a cylinder of Ti MMC 2.5" diameter in dry machining conditions. The machining speed was chosen to exceed the manufacturer's recommendation in order to see and check the tool cutting speed limits (80 m/min). The cutting depth was kept at 0.15mm and the cutting feed was 0.1mm.

The aim of this experiment was to find out the relation between AE and tool wear. During the test, every time when cutting length reached 10mm, the machine was stopped to manually measure the tool wear parameter ( $VB_B$ ). Figure 4 shows one example of a raw AE signal  $AE_{raw}$  directly from AE sensor. During the first 5~8s, the cutting tool is approaching the workpiece and gradually reaching the cutting depth. After 30s, the cutting tool leaves the surface of workpiece.



The middle period is the steady cutting period, which contains the most useful information for tool wear condition investigation. In the experiment, five AE signal sets were recorded according to different cutting sections: 0 ~ 10mm, 10 ~ 20mm, 20 ~ 30mm, 30 ~ 40mm and 40 ~ 50mm. The interval output of type-2 fuzzy modelling provides the information of uncertainty in AE estimations. The sufficient information from AE uncertainty scheme can be used to make decision or investigate tool condition so as to enhance the reliability of tool wear estimation. By applying type-2 fuzzy logic to AE based tool conditions monitoring, it is possible to automate on-line diagnosis of cutting tool condition [23].

This paper focus on filtering and capturing the uncertainty by a second order type-2 TSK fuzzy approach on raw RMS amplitudes of AE signal (AErms) during one 10mm cutting length in turning process. AErms signal from the AE sensor, shown in Fig. 6.3, was processed by the designed type-2 fuzzy logic filter to demonstrate its effectiveness.

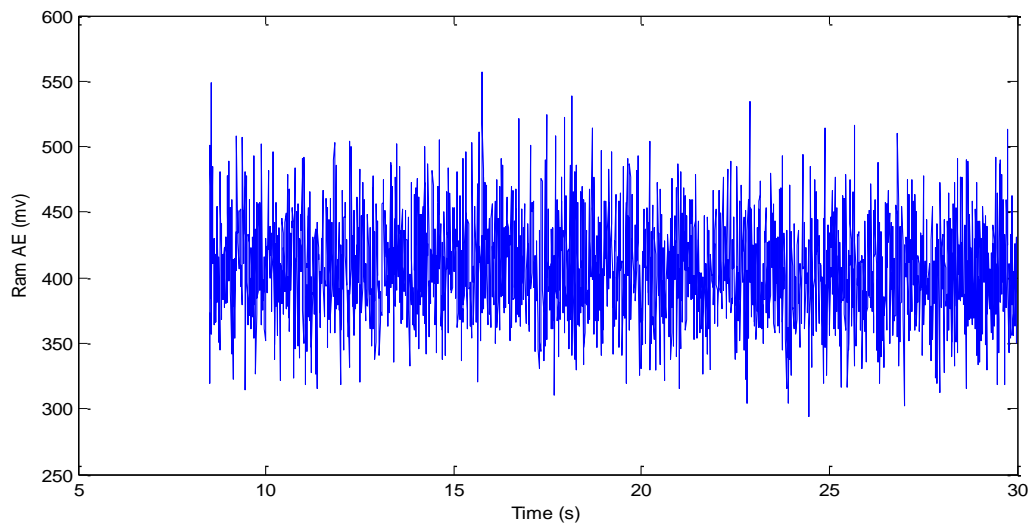


Fig. 6.3 AErms signal

For the second order IT2 TSK approach, there are 11 rules used to estimate the uncertainty in the AE signal and their antecedents are shown in Fig. 6.4. The spreading percentages for 11 clusters are listed in Table 6.1 and the spread percentage for consequent parameters is 2%. The AE signal modeling result is shown in Fig. 6.5. The difference between the upper boundary  $\overline{AE}$

(dashed curve) and the lower boundary  $\underline{AE}$  (dotted curve) identifies the uncertainty of AE signal at every moment. The overall identified AE signal value  $AE_{fuzzy}$  is shown as solid curve.

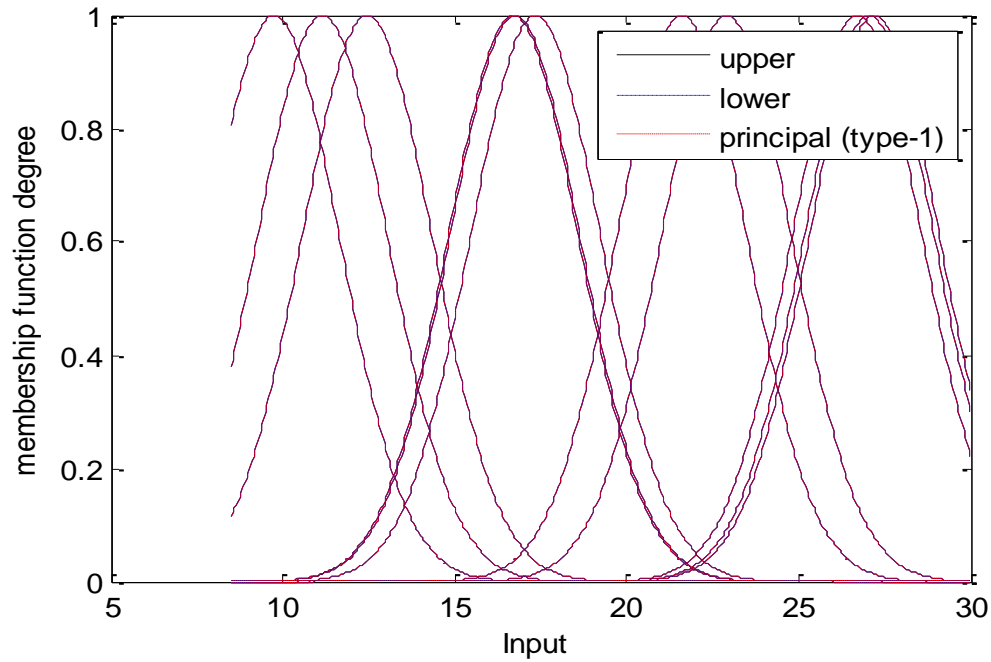


Fig. 6.4 Antecedents for second order IT2 TSK approach

Table 6.1 Spreading percentage of cluster centers for second order IT2 FLS

Rule Number	Spreading Percentage of Cluster Center
1	0,00141%
2	0,00951%
3	0,00883%
4	0,00437%
5	0,00835%
6	0,00325%
7	0,00368%
8	0,00795%
9	0,00099%
10	0,00952%
11	0,00001%

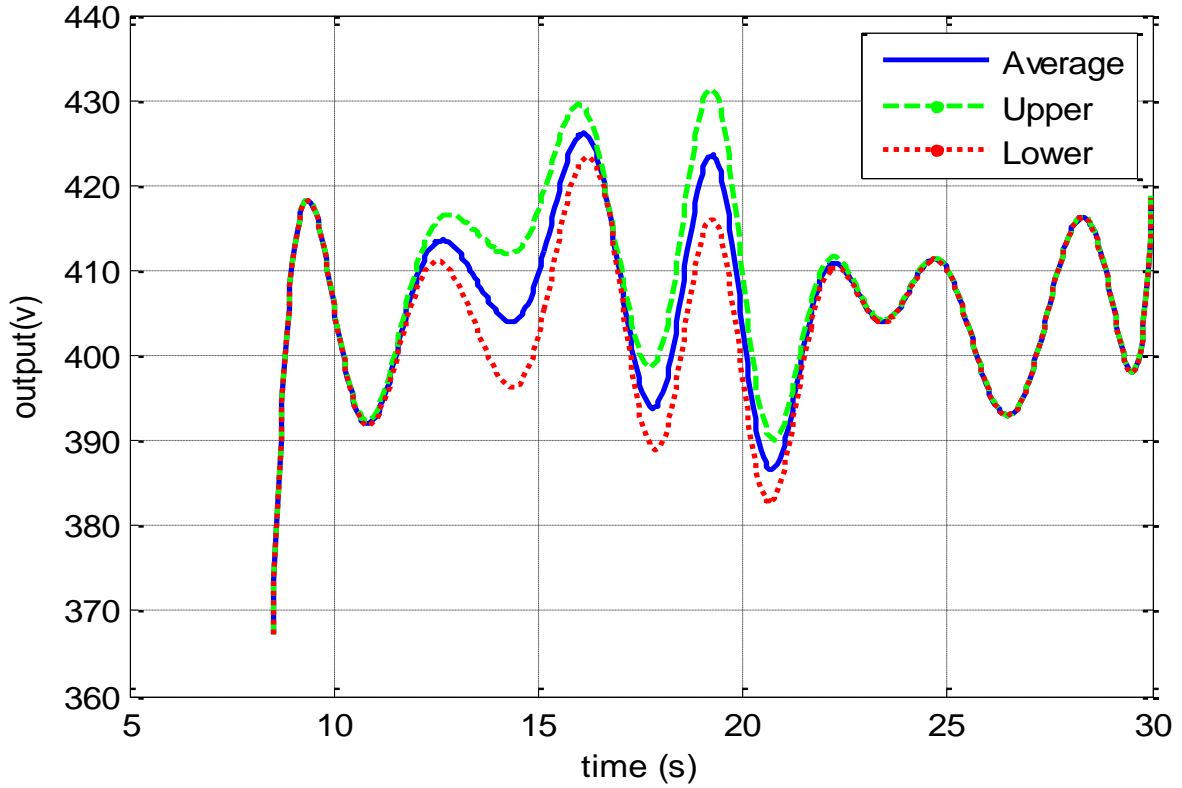


Fig. 6.5 AE signal approached by second order Type-2 TSK FLS

To demonstrate the advantage and efficiency of the second order IT2 FLS, the maximum and minimum difference between  $\overline{AE}$  and  $\underline{AE}$ ,  $\overline{AE}$  and  $\underline{AE}$ ,  $\underline{AE}$  and  $AE_{fuzzy}$  in modeling results are compared with that from first order IT2 fuzzy approach [22]. The first order IT2 TSK approach used 24 rules to identify AE signal and its antecedents are shown in Fig. 6.6. The approach result is shown in Fig. 6.7. The spread percentage for consequent parameters is 2%. The spreading percentages for 24 clusters are listed in Table 6.2.

From Table 6.3, it is observed that even the second order IT2 TSK FLS has less rules than that of first order FLS, it can modeling the AE signal with similar RMSE. This proves that a high order IT2 TSK rules can not only has the capability of an IT2 TSK FLS – handle uncertainties within FLS, but also it is able to reduce drastically the number of rules needed to perform the approximation, and improve transparency and interpretation in many high dimensional situations.

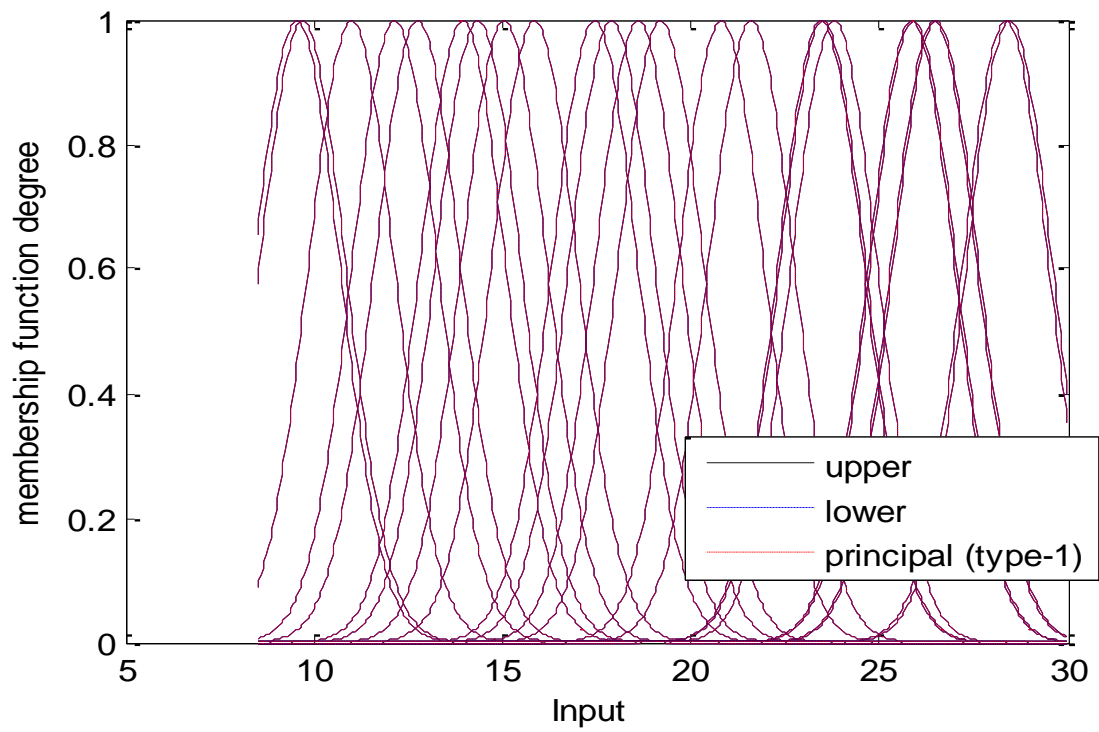


Fig. 6.6 Antecedents for first order IT2 TSK approach

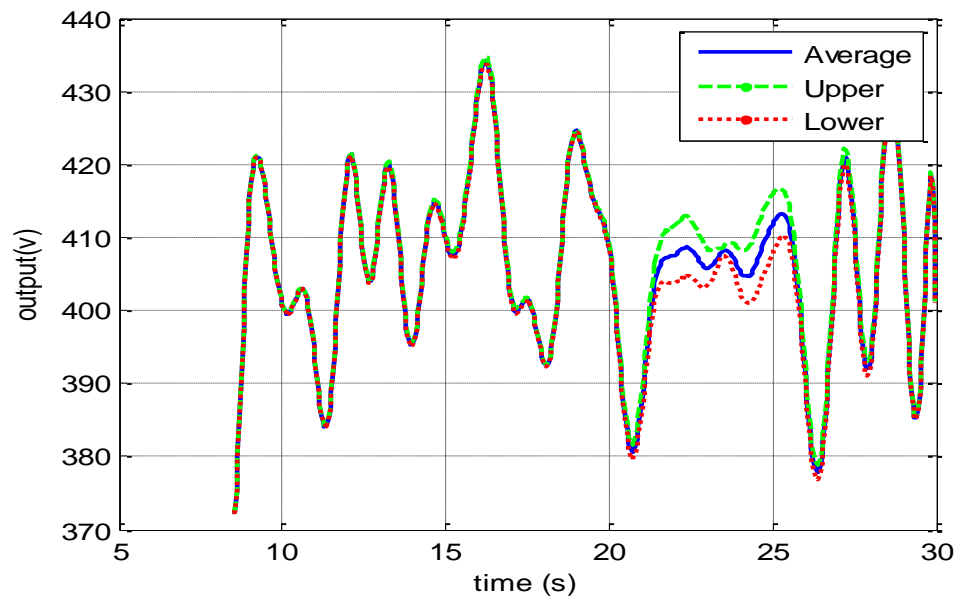


Fig. 6.7 AE signal approached by first order Type-2 TSK FLS

Table 6.2 Spreading percentage of cluster centers for first order IT2 FLS

Rule Number	Spreading Percentage of Cluster Center
1	0,00458%
2	0,00722%
3	0,00339%
4	0,00401%
5	0,00527%
6	0,00894%
7	0,00778%
8	0,00069%
9	0,00279%
10	0,00379%
11	0,00865%
12	0,00420%
13	0,00240%
14	0,00598%
15	0,00479%
16	0,00899%
17	0,00935%
18	0,00818%
19	0,00709%
20	0,00743%
21	0,00900%
22	0,00065%
23	0,00336%
24	0,00004%

The uncertainty assessed by type-2 fuzzy estimation can originate from several possible sources:

- Uncertainties from the machine itself: the structural compliance of the links, joints and drive components of the machine results in the oscillatory behaviour of the machine link motion and the driving torque profile;
- Uncertainties from the AE sensors: sensitivity to the environment change such as temperature and humidity, the circuit noise, the locating error of the sensors, etc;

- Uncertainties from the data processing system: the type-2 TSK filtering algorithm, because of need of identification of the spreading parameters, etc.

Table 6.3 Comparison of first order TSK approach and second order approach

Modelling Results	IT2 TSK approach	
	First order	Second order
Number of rules	24	11
RMSE (mv)	35.9213	36.6639
Max ( $ \overline{AE} - \underline{AE} $ )	6.7561	1.1474
Min ( $ \overline{AE} - \underline{AE} $ )	0.0113	0.0014
Max ( $ \overline{AE} - AE_{fuzzy} $ )	7.0735	0.6054
Min ( $ \overline{AE} - AE_{fuzzy} $ )	0.0540	0.0193
Max ( $ \underline{AE} - AE_{fuzzy} $ )	13.8296	0.5420
Min ( $ \underline{AE} - AE_{fuzzy} $ )	0.0652	0.0001

## 6.6 Conclusion

This paper presented the architecture, inference engine and a design method of the generalized IT2 TSK FLS and a design method of high order IT2 TSK FLS in which the antecedent or consequent membership functions are type-2 fuzzy sets and its consequent part is a high order polynomial function. From the experimental study, it is proven that high order IT2 TSK FLS not only can handle uncertainties within FLS, even measurement uncertainties, but also has the capability to overcome the problem of dimensionality.

The experimental acoustic emission signal modelling using a second order IT2 FLS demonstrates the advantage and efficiency of high order IT2 FLS. Estimation of uncertainty of AE using IT2 fuzzy system could be of great value to a decision maker and be used to investigate tool wear condition during machining process.

Type-2 fuzzy logic approach provides the possibility to indicate the uncertainties in manufacturing process to monitor an automated process, which could be crucial in maintaining

high production quality. The experimental study in this paper can be used in high precision manufacture for uncertainty estimation in machining and on-line tool condition monitoring development.

One limitation of this paper is only one simple single-input-single-output example is conducted to verify the performance of the high order type-2 FLS. More examples could be added in the future to draw a convincing conclusion.

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## Reference

1. Zadeh, L. A. (1965) Fuzzy Sets. Information and Control 8:338-353
2. Zadeh, L. A. (1975) The Conception of A Linguistic Variable and Its Application in Approximate Reasoning – I. Information Science 8:199-249
3. Mimmato M. and Tanaka K. (1976) Some properties of fuzzy sets of type-2. Information and Control, 31:312-340
4. Yager R. R. (1980) Fuzzy Subset of Type-II in Discussion. Journal of Cybernetics 10:137-159
5. Mimmato M. and Tanaka K. (1981) Fuzzy sets of type-2 under algebraic product and algebraic sum. Fuzzy Sets and Systems 5:277-290
6. Dubais D. and Prade H. (1980) Fuzzy Sets and Systems: Theory and Applications. Academic, New York.
7. Karnik N. N. and Mendel J. M. (2001) Centroid of a type-2 fuzzy set. Information Sciences 132:195-220.
8. Karnik N. N. and Mendel J. M. (2001) Operations on type-2 fuzzy sets. Fuzzy Sets and Systems 122(2):327-48

9. Karnik N. N., Mendel J. M. and Liang Q. (1999) Type-2 fuzzy logic systems. *IEEE Transactions Fuzzy System*.7: 643-658
10. Liang Q. and Mendel J. M. (2000) Interval type-2 fuzzy logic systems: Theory and design. *IEEE Transactions Fuzzy System* 8:535-550
11. Mendel J. M. and John R. I. (2002) Type-2 Fuzzy Sets Made Simple. *IEEE Transaction On Fuzzy System* 10(2):117-127
12. Mendel J. M. (2001) *Uncertain Rule-Based Fuzzy logic Systems – Introduction on New Directions*. Prentice hall PTR, upper saddle river, NJ
13. Liang Q. and Mendel J. M. (1999) An Introduction to Type-2 TSK Fuzzy Logic Systems. In: 1999 IEEE International Fuzzy Systems Conference Processing, Seoul, Korea
14. Dereli T, Baykasoglu A, Altun K, Durmusoglu A, Turksen B. (2010) Industrial applications of type-2 fuzzy sets and systems: A concise review, *Computers in industry* 62(2):125-137
15. Bellman R., (1961). *Adaptive Control Processes: A Guide Tour*. Princeton University Press, Princeton, NJ
16. Ren Q., Baron L. and Balazinski M., (2008) High order type-2 fuzzy logic system. In: *The 27th North American Fuzzy Information Processing Society Annual Conference*, New York, United states.
17. Demirli K. and Muthukumaran P. (2000) Higher Order Fuzzy System Identification Using Subtractive Clustering. *Journal of Intelligent and Fuzzy Systems*, 9:129-158.
18. Buckley J. J. (1993) Sugeno-type controllers are universal controllers. *Fuzzy Sets and Systems* 53:299–303
19. Ren Q., Baron L. and Balazinski M. (2006) Type-2 Takagi-Sugeno-Kang Fuzzy Logic Modelling using Subtractive Clustering. In: *The 25th North American Fuzzy Information Processing Society Annual Conference*, Montreal, Canada



20. Ren Q., Baron L., Balazinski M. and Jemielniak K. (2011) TSK Fuzzy Modeling for Tool Wear Condition in Turning Processes: An Experimental Study. *Engineering Applications of Artificial Intelligence* 24(2):260-265
21. Ren Q., Baron L. and Balazinski M. (2011) Fuzzy Identification of Cutting Acoustic Emission with Extended Subtractive Cluster Analysis. *Nonlinear Dynamics* DOI: 10.1007/s11071-011-0173-5
22. Ren Q., Baron L. and Balazinski M., (2009) Application of Type-2 Fuzzy Estimation on Uncertainty in Machining: An Approach on Acoustic Emission during Turning Process. In: *The 28th North American Fuzzy Information Processing Society Annual Conference*, Cincinnati, Ohio, USA
23. Ren Q., Baron L., and Balazinski M., (2011) Type-2 Fuzzy Modeling for Acoustic Emission Signal in Precision Manufacturing. *Modeling and Simulation in Engineering* 2011(696947) DOI:10.1155/2011/696947

## **GENERAL DISCUSSIONS**

In the past years, my directors and I had chances to attend several conferences and meet experts from different domains in the type-2 FL community. These experiences helped us to improve our knowledge on advances of type-2 FS and FLS.

This thesis is mainly composed by the three published journal papers with the application on fuzzy AE identification in high precision hard turning process. Actually, during this long-term Ph.D. research period, we also applied the type-1 and type-2 TSK fuzzy identification algorithm to some other applications in mechanical engineering. All of them are published in conference proceedings, journals or as book chapters.

Applications of type-1 and type-2 fuzzy logic and future perspectives based on our previous research and on-going studies are listed as followings.

### **Applications of Type-1 Fuzzy Logic**

- Tool Condition Monitoring Using the TSK Fuzzy Approach Based on Subtractive Clustering Method (Ren, Baron, Balazinski & Jemelniak, 2008; Ren, Baron, Balazinski & Jemelniak, 2011). It is an experimental study for turning process in machining by using Takagi-Sugeno-Kang (TSK) fuzzy modeling to accomplish the integration of multi-sensor information and tool wear information. It generates fuzzy rules directly from the input-output data acquired from sensors, and provides high accuracy and high reliability of the tool wear prediction over a wide range of cutting conditions. The experimental results show its effectiveness and satisfactory comparisons relative to other artificial intelligence methods.
- Joint friction identification for robots using TSK fuzzy system based on subtractive clustering (Qin, Ren, Baron, Balazinski & Birglen, 2008). The proposed approach can provide accurate prediction of the joint friction despite the nonlinearity of the friction and measurement uncertainty. Simulation results show the effectiveness and convenience of the method.

- fuzzy cutting force modelling in micro-milling using subtractive clustering for learning evaluation (Ren, Baron, Balazinski & Jemelnik, 2010(b)). Subtractive clustering, combined with the least-square algorithm, identifies the fuzzy prediction model directly from the information obtained from the sensors. In the micro-milling experimental case study, two fuzzy models learned through different evaluation strategies are tested.

## **Applications of Type-2 Fuzzy Logic**

- Identification of rigid-body dynamics of robotic manipulators using type-2 fuzzy logic filter (Ren, Qin, Baron, Birglen & Balazinski, 2007). Subtractive clustering based type-2 Takagi-Sugeno-Kang (TSK) fuzzy logic process is used as a fuzzy filter to treat acceleration data for the purpose of obtaining the rigid-body dynamical parameters of robotic manipulators. Experimental results show the effectiveness of this method, which not only provides good accuracy of prediction of the rigid-body dynamical parameters of robotic manipulators, but also assesses the uncertainties associated with the modeling process and with the outcome of the model itself.
- Uncertainty prediction for tool wear condition in turning process using type-2 fuzzy approach (Ren, Baron & Balazinski, 2009 (a)). Type-2 fuzzy approach not only provides high reliability of the tool wear prediction over a range of cutting condition, but also assesses the information of uncertainty of the estimation. The magnitude and direction of uncertainties in the machining process are described explicitly to increase the credibility of assessments.
- Modelling of dynamic micromilling cutting forces using type-2 fuzzy rule-based system (Ren, Baron, Jemelnik & Balazinski, 2010). The type-2 fuzzy estimation not only filters the noise and estimates the instantaneous cutting force in micromilling using observations acquired by sensors during cutting experiments, but also assesses the uncertainties associated with the prediction caused by the manufacturing errors and signal processing. Moreover, the interval output of the type-2 fuzzy system gives very useful information to machine tool controllers in order to maximize material removal while controlling tool wear or tool failure to maintain part quality specifications.

- Fuzzy uncertainty estimation in machining is a developing trend for high precision manufacture research. A reliable on-line type-2 fuzzy TCM base on acoustic emission (AE), along with other cutting parameters, can be developed for high precision machining (Ren, Baron, Balazinski & Jemielniak, 2012). An original research paper on this subject was submitted to Elsevier journal “Information Science” (Ren, Balazinski, Baron & Jemielniak, 2011) and had been feed-backed to modify for acceptance. In this paper, a reliable type-2 fuzzy tool condition monitoring system based on micromilling multiple acoustic emission signal features is proposed. Type-2 fuzzy logic system is used as not only a powerful tool to model acoustic emission signal, but also a great estimator for the ambiguities and uncertainties associated with the signal. In the experimental study, a total of 27 AE signal features (SFs) are identified by type-2 FLSs. Depend on the estimation of RMSE and variations in modeling results of all signal features, reliable ones are selected and integrated to estimate of cutting tool life estimation. The experiment shows that the type-2 fuzzy tool life estimation is in accordance with cutting tool wear state in the micromilling process. Key components of the proposed architecture of type-2 fuzzy analysis based TCM are shown in Fig. G.1.

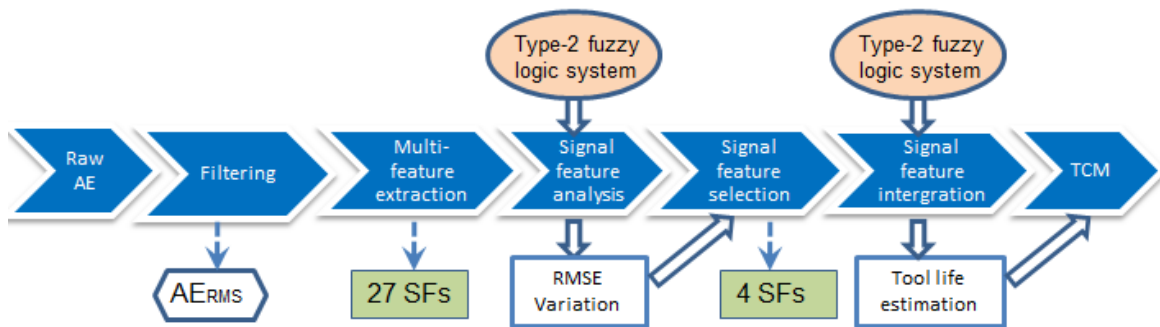


Fig. G.1 Architecture of type-2 fuzzy tool condition monitoring system

## Perspectives

- As a universal approximation tool, traditional (type-1) FLSs has been widely used for different aspects of mechanical engineering. It is believed that type-2 FL moves in

progressive ways where type-1 FL is eventually replaced or supplemented by T2 FL (Mendel, 2007). Type-2 FL will be widely applied to manufacturing, design and control.

- It is more difficult to interpret a higher-order consequent part than a zero- or first-order consequent part in a fuzzy system. This thesis only conducts one simple SISO example to verify the performance of the high order type-2 FLS. More examples could be added in the future to draw a convincing conclusion.
- Because type-2 fuzzy logic is a very new research subject, there are still a lot of challenges to overcome.
  - The performance of an IT2 FLS depends heavily on its learning algorithm. In this thesis, the type-1 and type-2 TSK FLSs are based on subtractive clustering method. Comparative analysis can be done with the FLSs from other structure identification approach and parameter learning algorithm.
  - Along with the idea of the more general kind of type-2 FS, Type-2 FLS, which is more robust to uncertainty, will be gradually developed. For example, with quasi type-2 FS developed on the basis of the  $\alpha$ -level representation for general T2 FSs (Lui, 2008; Mendel, Liu & Zhai, 2009), quasi type-2 FLS can provide more accurate estimation on uncertainties.

## **CONCLUSIONS**

This thesis presented type-1 TSK FLS, type-2 TSK identification algorithm based on subtractive clustering method and proposes high order type-2 TSK FLS. The experimental studies on AE in a turning process showed the benefits of type-2 fuzzy modeling and advantage of higher order type-2 modeling.

### **Conclusion summaries**

This thesis is composed by three published papers, in the order as type-1 TSK FLS, first order interval type-2 FLS and high order Interval FLS, based one experimental study – uncertainty estimation for acoustic emission signal for a precision turning process.

The thesis well compared the three system based on the same experimental study, and show the differences between the type-2 FLS and its type-2 counterpart, and these between first order and second order type-2 system.

The research summaries are listed as followings.

- Type-2 FLS can be used as a powerful tool to model machining dynamics from the vague information as its type-1 counterpart. Type-2 fuzzy modelling performs better but requires longer computational time.
- Type-2 FLS is a great estimator for the ambiguities and uncertainties in the machining process. The variations estimated from AE signal correspond to the period of cutting tool wear or failure.
- Type-2 fuzzy uncertainty modeling on cutting dynamics (AE, cutting force) can estimate cutting tool state during the cutting process and can be used to develop type-2 fuzzy logic based tool condition monitoring system (TCM);
- It is possible to predict variability in a process in real-time by using type-2 fuzzy logic.

- High order interval type-2 FLS not only can handle uncertainties within FLS, but also has the capability to overcome the problem of curse of dimensionality.
- The application of type-2 FL on uncertainty estimation in high precision machining has great meaning for continuously improvement in product quality, reliability and manufacturing efficiency in machining industry.

## Research Contributions

The main contributions of this thesis are listed as followings.

- Applied the type-2 TSK identification algorithm based on subtractive clustering method on uncertainty estimation in machining processes.
- Proved that type-2 fuzzy system has better performance than its type-1 counterpart;
- Proposed the generalized interval type-2 FLS.
- Established the theoretical basics for high order type-2 FLS.
- Proved that a second order type-2 FLS can reduce the number of rules and has similar modeling result as that of a first order system. .

## REFERENCES

- Abiyev R. H., Kaynak O, Alshanableh T & Mamedov F. (2011). A type-2 neuro-fuzzy system based on clustering and gradient techniques applied to system identification and channel equalization. *Applied Soft Computing*, 11(1), 1396-1406
- Al-Wedyan H., Demirli K. & Bhat R. (2001). A technique for fuzzy logic modeling of machining process”, *Proceedings Joint 9th IFSA World Congress and 20th NAFIPS International Conference* (vol. 5, pp.3021-3026).
- American National Standards Institute (1997). *American National Standard for Expressing Uncertainty—U.S. Guide to the Expression of Uncertainty in Measurement*, ANSI/NCSL Z540-2-1997..
- Babuska R. (1998). *Fuzzy Modeling for Control*. Boston: Kluwer Academic Publishers.
- Baker M. (2005). Does chip formation minimize the energy. *Computational Materials Science*, 33, 407–418.
- Barkat, S., Tlemçani, A. & Nouri, H. (2011). Noninteracting Adaptive Control of PMSM Using Interval Type-2 Fuzzy Logic Systems. *IEEE Transactions on Fuzzy Systems*, 19(5), 925-926.
- Bellman R. (1961). *Adaptive Control Processes: A Guide Tour*. Princeton, NJ: Princeton University Press.
- Bezdek J. (1974). Cluster Validity with Fuzzy Sets. *Journal of Cybernetics*, 3, 58-71.
- Bezdek J., Hathaway R., Sabin M. & Tucker W. (1987). Convergence Theory for Fuzzy C-means: Counter Examples and Repairs. In J. Bezdek (Ed.), *The Analysis of Fuzzy Information* (vol. 3, chapter 8). Boca Raton, FL: CRC Press.
- Biglarbegian M., Melek W. W. & Mendel J. M. (2010). On the Stability of Interval Type-2 TSK Fuzzy Logic Control Systems. *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, 10(3) , 798-818.



- Biglarbegian M., Melek W. W. & Mendel J. M. (2011). Design of Novel Interval Type-2 Fuzzy Controllers for Modular and Reconfigurable Robots: Theory and Experiments. *IEEE Transactions on Industrial Electronics*, 58(4) 1371-1384.
- Bryan J. B. (1990). International status of thermal error research, *Annals CIRP*, 39(2), 645–656.
- Castroa H. F. & Burdekin M. (2005). Evaluation of the measurement uncertainty of a positional error calibrator based on a laser interferometer. *International Journal of Machine Tools & Manufacture*. 45, 285–291.
- Chatterjee S. (1997). An assessment of quasi-static and operational errors in NC machine tools. *J. Manuf. Syst.*, 16(11), 59–68.
- Chen C. S. (2011). Supervisory adaptive tracking control of robot manipulators using interval type-2 TSK fuzzy logic system. *Control Theory & Applications*, 5(15), 1796-1807.
- Chiu S. L. (1994). Fuzzy Model Identification Based on Cluster Estimation, *Journal of Intelligent & Fuzzy Systems*, 2, 267–278.
- Chiu S. L. (1997). Extracting Fuzzy Rules from Data for Function Approximation and Pattern Classification. In D. Dubois, H. Prade, R.R. Yager (Eds.), *Fuzzy Information Engineering: a Guide Tour of Applications* (pp. 149–162). New York: Wiley.
- Demirli K. & Muthukumaran P. (2000). Higher Order Fuzzy System Identification Using Subtractive Clustering. *Journal of Intelligent and Fuzzy Systems*, 9, 129-158.
- Dereli, T., A. Baykasoğlu, K. Altun, A. Durmusoğlu & I. B. Turksen (2011). Industrial applications of type-2 fuzzy sets and systems: A concise review. *Computers in Industry*, 62(2), 125-137.
- Dubois D. & Prade H. (1978). Operations on Fuzzy Numbers. *International Journal on Systems Science*, 9, 613-626.
- Dubois D. & Prade H. (1979). Operations in A Fuzzy-Valued Logic. *Information and Control*, 43, 224-240.
- Dubois D. & Prade H. (1980). *Fuzzy Sets and Fuzzy Systems: Theory and Applications*. NY: Academic press.

- Emami M. R., Truksen I. B. & Goldenberg A. A.. (1998). Development of A Systematic Methodology of Fuzzy Logic Modeling. *IEEE Transaction on Fuzzy Systems*, 6, 346–361.
- Fadali M. S, Jafarzadeh S. & Nafeh A. (2010). Fuzzy TSK Approximation Using Type-2 Fuzzy Logic Systems and Its Application to Modeling a Photovoltaic Array. *Proceeding of 2010 American Control Conference Marriott Waterfront, Baltimore, MD, USA* (pp. 6454-6459).
- Füssel D., Ballé P. & Isermann R. (1997). Closed Loop Fault Diagnosis Based on A Nonlinear Process Model and Automatic Fuzzy Rule Generation. *Proceedings of 4th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, The University of Hull, UK*.
- Galan P. (2004). Temperature control: PID and fuzzy logic. *Control Engineering Europe*, 36.
- Garcia, J. C. F. (2009). An evaluative Interval Type-2 TSK Fuzzy Logic System for volatile time series identification. *Proceeding of IEEE International Conference Systems, Man and Cybernetics, 2009. SMC 2009. San Antonio, TX* (pp. 666 – 671).
- Gupta M. M. (2005). Development of higher-order neural units for control and pattern recognition. *Proc. of NAFIPS 2005* (pp. 395-400).
- Hagras H. (2007). Type-2 FLCs: a new generation of fuzzy controllers. *IEEE Computational Intelligence Magazine*, 2(1), 30-43.
- Hatamura Y., Nagao T., Mitsuishi M., Kato K., Taguchi S. & Okumura T. (1993). Development of an intelligent machining center incorporating active compensation for thermal distortion. *Annals CIRP*, 42(1), 549–552.
- Herrera L. J., Pomares H., Rojas I., Valenzuela O. & Prieto A. (2005). TaSe, a Taylor series-based fuzzy system model that combines interpretability and accuracy. *Fuzzy Sets and Systems*, 153, 403–427.
- Ikawa N. Shimada S. & Tanaka H. (1992). Minimum thickness of cut in micromachining. *Nanotechnology*, 3, 6-9.
- International Organization for Standardization (1986). *Guide to the Expression of Uncertainty in Measurement*.

- Jantunen E. (2006). Diagnosis of tool wear based on regression analysis and fuzzy logic. *IMA Journal of Management Mathematics*, 17(1), 47-60.
- Jetly S. (1984). Measuring cutting tool wear on-line: some practical considerations. *Manufacturing Engineering*, July, 55-60.
- Johansen T. & Foss B. (1995). Identification of Non-Linear System Structure and Parameters Using Regime Decomposition. *Automatic*, 31(2), 321-326.
- John R. I. (1998). Type-2 Fuzzy Sets: An Appraisal of Theory and Applications. *International Journal of Uncertainty, Fuzziness and Knowledge Based Systems*, 6(6), 563-576.
- John R. I. (2002). Embedded Interval Valued Type-2 Fuzzy Sets", *Proceeding of the 2002 FUZZ-IEEE international conference* (vol. 2, pp.1316-1320).
- John R. & Coupland S. (2007). Type-2 fuzzy logic: A historical view. *IEEE Computational Intelligence Magazine*, 2(1), 57-62.
- Karnik N. N. & Mendel J. M.(1998). *Introduction to Type-2 Fuzzy Logic Systems*, University of Southern California. <http://sipi.usc.edu/-Mendel/report>.
- Karnik N. N., Mendel J. M. & Liang Q. (1999). Type-2 fuzzy logic system. *IEEE Transaction on Fuzzy Systems*, 7, 643-658.
- Kayacan E, Oniz Y, Aras A. C., Kaynak O. & Abiyev R. (2011). A servo system control with time-varying and nonlinear load conditions using type-2 TSK fuzzy neural system. *Applied Soft Computing*, 11(8), 5735-5744.
- Kim K., Kyung K. M., Park C.W., Kim E. & Park M. (2004). Robust TSK Fuzzy Modeling Approach Using Noise Clustering Concept for Function Approximation. *Computational and Information Science* (vol.3314). Springer Berlin: Heidelberg.
- Liang Q. & Mendel J. M. (1999). An Introduction to Type-2 TSK Fuzzy Logic Systems. *1999 IEEE International Fuzzy Systems Conference Proceeding, Seoul, Korea* (vol. 2, 1534 - 1539).
- Liang Q. & Mendel J. M. (2000). Equalization of nonlinear time-varying channels using type-2 fuzzy adaptive filters. *IEEE Transactions on Fuzzy Systems*, 8(5), 551-563.

- Lui F. (2008). An efficient centroid type-reduction strategy for general type-2 fuzzy logic system, *Information Sciences*, 178(9) 2224-2236.
- Mamdani E. H. & Assilian S. (1974). Applications of Fuzzy Algorithms for Control of Simple Dynamic Plant. *Proc. Inst. Elec. Eng.* (vol. 121, 1585-1588).
- Mamdani E. H. & Assilian S. (1997). An Experiment in Linguistic Synthesis with A Fuzzy Logic Controller. *International Journal Man-Machine Study*. 7, 1–3.
- Mendel J. M. (2001). *Uncertain Rule-Based Fuzzy Logic Systems – Introduction and New Directions*. Upper Saddle River, NJ: Prentice Hall PTR.
- Mendel J. M., Hagens H., John R. I. (2006). *Standard background material about interval type-2 fuzzy logic systems that can be used by all authors.* <http://ieeecs.org/files/standards.t2.win.pdf>.
- Mendel J. M. & John R. I. (2001). A Fundamental Decomposition of Type-2 Fuzzy Sets. *Proceeding of IFSA world congress and 20<sup>th</sup> NAFIPS international conference* (vol. 4, 1896-1901).
- Mendel J. M. & John R. I. (2002). Type-2 Fuzzy Sets Made Simple. *IEEE Transaction on Fuzzy System*, 10(2), 117-127.
- Mendel J. M., Liu F. & Zhai D. (2009).  $\alpha$ -Plane Representation for Type-2 Fuzzy Sets: Theory and Applications, *IEEE Trans. On Fuzzy systems*, 17(5) 1189-1205.
- Mendez G. M., Castillo O. & Hernandez M. M. (2006). First-order interval type-1 non-singleton type-2 TSK fuzzy logic systems. *Proceeding of MICAI 2006: Advances in Artificial Intelligence. 5th Mexican International Conference on Artificial Intelligence* (Vol. 4293, pp. 81-89).
- Mendez G. M. & Hernandez M. D. L. A. (2010). Interval type-1 non-singleton type-2 fuzzy logic systems are type-2 adaptive neuro-fuzzy inference systems. *International Journal of Reasoning-based Intelligent Systems*, 2(2), 95-99.
- Mizumoto M. & Tanaka K. (1976). Some Properties in Fuzzy Sets of Type-2. *Information and Control*, 31, 312-340.

- Mizumoto M. & Tanaka K. (1981). Fuzzy Sets of Type-2 under Algebraic Product Algebraic Sum. *Fuzzy Sets and Systems*, 5, 277-290.
- Nandi A. K. & Davim J. P. (2009). A study of drilling performances with minimum quantity of lubricant using fuzzy logic rules. *Mechatronics*, 19(2), 218-232.
- Nieminen J. (1997). On the Algebraic Structure of Fuzzy Sets of Type-2. *Kybernetika*, 13(4).
- Ogata K. (1970). *Modern Control Engineering*. Englewood cliffs, N. J.: Prentice hall.
- Pedrycz W. (1984). Identification in Fuzzy Systems. *IEEE Transaction on Fuzzy Systems*, 14, 361–366.
- Qin, Z. K., Ren Q., Baron L. Balazinski M. & Birglen L. (2008). Joint Friction Identification for Robots Using TSK Fuzzy System Based on Subtractive Clustering. *Proceeding of The 27th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2008)*, New Yoke, NY, USA (pp.1-6)
- Ramesh R., Mannan M. A. & Poo A. N. (2002). Support vector machines model for classification of thermal error in machine tools. *International Journal of Advanced Manufacturing Technology*, 20, 114–120.
- Ren Q. (2006). *Type-2 Takagi-Sugeno-Kang Fuzzy Logic Modeling using Subtractive Clustering*. M.Sc.A. École Polytechnique de Montréal, Qc., Canada.
- Ren Q., Baron L. & Balazinski M. (2006). Type-2 Takagi-Sugeno-Kang Fuzzy Logic Modeling using Subtractive Clustering. *Procedding of 25th International Conference of the North American Fuzzy Information Processing Society NAFIPS'06*, Montreal, Canada (1-6).
- Ren Q., Qin Z.-K, Baron L. Birglen L. & Balazinski M. (2007). Identification of rigid-body dynamic of robotic manipulators using type-2 fuzzy logic filter. *Proceedings of 26th International Conference of the North American Fuzzy Information Society NAFIPS'07* (389-392).
- Ren Q., Baron L. & Balazinski M. (2008). High Order Type-2 TSK Fuzzy Logic System. *Proceeding of The 27th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2008)*, New Yoke, NY, USA, (1-6)

- Ren, Q., Baron, L., Balazinski, M. & Jemelniak, K. (2008). Tool Condition Monitoring Using the TSK Fuzzy Approach Based on Subtractive Clustering Method. *News Frontiers in Applied Artificial Intelligence*, Springer Berlin, 52-61.
- Ren Q., Baron L. & Balazinski M. (2009 (a)). Uncertainty Prediction for Tool Wear Condition Using Type-2 TSK Fuzzy Approach. *Proceeding of the 2009 IEEE International Conference on Systems, Man, and Cybernetics (IEEE-SMC 2009), San Antonio, TX, USA* (pp. 666 - 671).
- Ren Q., Baron L. & Balazinski M. (2009(b)), “Type-2 TSK Fuzzy Filtering and Uncertainty Estimation for Acoustic Emission in Precision Manufacturing”, *Proceedings of the IASTED International Conference Computational Intelligence (IASTED-CI 2009), Honolulu, Hawaii, USA* (pp.170-175).
- Ren Q., Baron L. & Balazinski M. (2009(c)), “Application of Type-2 Fuzzy Estimation on Uncertainty in Machining: An Approach on Acoustic Emission during Turning Process”, *The 28th North American Fuzzy Information Processing Society Annual Conference (NAFIPS 2009), Cincinnati, Ohio, USA* (pp. 1-6).
- Ren Q., Baron L. Balazinski M. & Jemielniak, K. (2010(a)). Acoustic emission signal feature analysis using type-2 fuzzy logic System. *Proceedings of 29th International Conference of the North American Fuzzy Information Society NAFIPS'10* (pp. 1-6).
- Ren Q., Baron L., Balazinski M. & Jemielniak K. (2010(b)). Fuzzy cutting force modelling in micromilling using subtractive clustering for learning evaluation. *Proceeding of 2010 CIRP 2nd international conference: process machine interactions (CIRP-PMI 2010), Vancouver, Canada*.
- Ren Q., Baron L. Jemielniak, K. & Balazinski M. (2010). Modeling of dynamic micromilling cutting forces using type-2 fuzzy rule-based system. *Proceeding of 2010 IEEE International Conference on Fuzzy Systems, Barcelona, Spain* (pp. 23131-23177).
- Ren Q., Balazinski M. & Baron L. (2011). Type-2 TSK Fuzzy Logic System and Its Type-1 Counterpart. *International journal of computer applications*, 20(6), 8-13.

- Ren Q., Baron L., & Balazinski M. (2011). Type-2 Fuzzy Modeling for Acoustic Emission Signal in Precision Manufacturing. *Modeling and Simulation in Engineering*, 696947, 12 pages. DOI:10.1155/2011/696947.
- Ren, Q., Baron, L., Balazinski, M. & Jemelniak, K. (2011). TSK fuzzy modeling for tool wear condition in turning processes: An experimental study. *Engineering Applications of Artificial Intelligence*, 24(2) 260-265.
- Ren Q., Baron L., Balazinski M. & Jemielniak K. (2011). Type-2 Fuzzy Tool Condition Monitoring System Based on Acoustic Emission in Micromilling. *information science*, submitted.
- Ren Q., Baron L. & Balazinski M. (2012). Fuzzy Identification of Cutting Acoustic Emission with Extended Subtractive Cluster Analysis. *Nonlinear Dynamics*, 67(4), 2599-2608.
- Ren Q., Balazinski M. & Baron L., (2012). High Order Interval Type-2 Takagi-Sugeno-Kang Fuzzy Logic System and Its Application in Acoustic Emission Signal Modeling in Turning Process. *International Journal of Advanced Manufacturing Technology*. DOI: 10.1007/s00170-012-3956-z
- Ren Q., Baron L. Balazinski M. & Jemielniak, K. (2012). Reliable Tool Life Estimation with Multiple Acoustic Emission Signal Feature Selection and Integration Based on Type-2 Fuzzy Logic. *Advances in Fuzzy Sets of Type-2: Theory and Applications*, Berlin: Springer-Verlag, submitted.
- Savkovic-Stevanovic J. B. (2004). The higher order multilevel fuzzy logic controller. *Chemical and Biochemical Engineering Quarterly*, 18(4), 345-52.
- Schmitz T. L. Jaydeep K., Kim N. H. & Abbas A. (2011). Uncertainty in Machining: Workshop Summary and Contributions. *Journal of Manufacturing Science and Engineering*, 133(5), 051009.1-051009.9.
- Shin Y. C. & Wei Y. (1992). A statistical analysis of positional errors of a multiaxis Machine tool. *Precision Eng.*, 14, 139–146.
- Song Q., Ma T. & Kasabov N. (2003). A Novel Generic Higher-order TSK Fuzzy Model for Prediction and Applications for Medical Decision Support. *Proceedings of the Eighth*

- Australian and New Zealand Intelligent Information Systems Conference (ANZIIS 2003)*, (pp. 241-245).
- Song Q., Ma T. & Kasabov N. (2005). Transductive Knowledge Based Fuzzy Inference System for Personalized Modeling. *Fuzzy Systems and Knowledge Discovery* (vol. 3614). Springer Berlin: Heidelberg.
- Sugeno M. & Kang G. (1986). Fuzzy Modeling and Control of Multilayer Incinerator. *Fuzzy Sets and Systems*, 18, 329–346.
- Sugeno M. & Kang G. (1988). Structure Identification of Fuzzy Model. *Fuzzy Sets and Systems*, 28(1), 15-33.
- Sugeno M. & Tanaka K. (1991). Successive Identification of A Fuzzy Model and Its Application to Prediction of A Complex System. *Fuzzy Sets and Systems*, 42, 315–334.
- Sugeno M. & Yasukawa T., (1993). A Fuzzy-Logic-Based Approach to Qualitative Modeling. *IEEE Transaction on Fuzzy Systems*, 1, 7–31.
- Swyt D. (2001). A. Length & Dimensional Measurements at NIST. *Journal of Research of the National Institute of Standards and Technology*, 106(1).
- Takagi T. & Sugeno M. (1985). Fuzzy Identification of Systems and Its Applications to Modeling and Control. *IEEE Transaction on Systems, Man, and Cybernetics*, 15(1), 116-132.
- Tanaka K., Yoshida H., Ohtake H. & Wang H. O. (2009). A Sum-of-Squares Approach to Modeling and Control of Nonlinear Dynamical Systems With Polynomial Fuzzy Systems. *IEEE Transaction on Fuzzy Systems*, 17(4), 911-922.
- Taylor B. N. & Kuyatt. C. E. (1994). *Guidelines for evaluating and expressing the uncertainty of NIST measurement results*. NIST Technical Note 1297, 20899. Gaithersburg, MD: National Institute of Standards and Technology.
- Theochaires J. B. (2006). A High-order Recurrent Neuro-fuzzy System with Internal Dynamics: Application to the Adaptive Noise Cancellation. *Fuzzy Sets and Systems*, 157, 471-500.



- Tong R. M. (1979). The Construction and Evaluations of Fuzzy Models”, in: M.M. Gupta, R.K. Rewgade, R.R. Yager (Eds.), *Advances in Fuzzy Set Theory and Applications* (pp.559–576). North-Holland: Amsterdam.
- Trojan G. J., Kiszka J. B., Gupta M.M., & Nikiforuk P.N. (1987). Solution of Multivariable Fuzzy Equation. *Fuzzy Sets and Systems*, 22, 271–279.
- Tsakonas, A. & Gabrys, B. (2011). Evolving Takagi-Sugeno-Kang fuzzy systems using multi-population grammar guided genetic programming. *Proceeding of International Conference on Evolutionary Computation Theory and Applications (ECTA'11), Paris, France*.
- Wang L. –X & Mendel J. M. (1992). Fuzzy Basis Functions, Universal Approximation, and Orthogonal Least Squares Learning. *IEEE Transaction on Neural Networks*, 3, 807-813.
- Wilhelm R. G. Srinivasan N. & Farabaugh F. (1997). Part form errors predicted from machine tool performance measurements. *Annals CIRP* , 46, 471–474.
- Wu C. Y. (1998). *Development of an ultrasonic virtual workpiece coordinate system*, Ph.D thesis, collage of engineering, the Pennsylvania state university.
- Wu D. & Tan W. W. (2006). A simplified type-2 fuzzy logic controller for real-time control. *ISA Transactions*, 45(4), 503-16.
- Xie S. Q. & Jamwal P. K. (2010). An iterative fuzzy controller for pneumatic muscle driven rehabilitation robot. *Expert Systems with Applications*, 38(7), 8128-8137.
- Yager R. R. (1980). Fuzzy Subset of Type-II in Discussion. *Journal of Cybernetics*, 10, 137-159.
- Yager R. R. & Filev D. P. (1992). *Approximate Clustering via Mountain Method*. Machine Intelligence Institute, Iona College, New Rochelle, NY.
- Yager R. R. & Filev D. P. (1994). *Essentials of Fuzzy Modeling and Control*. New York: Wiley.
- Yao L., Jiang Y. L. & Xiao J. (2011). Short-Term Power Load Forecasting by Interval Type-2 Fuzzy Logic System. *Information Computing and Applications*, 2444(7), 575-582.
- Yildirim, M. T., Basturk, A. & Yuksel, M.E. (2008). Impulse Noise Removal From Digital Images by a Detail-Preserving Filter Based on Type-2 Fuzzy Logic. *IEEE Transactions on Fuzzy Systems*, 16(4), 920-928.

- Yu N. & N. Y. Zhang (2005). Fuzzy sliding-mode control for higher-order SISO nonlinear systems. *Journal of Tsinghua University (Science and Technology)*, 45(10).
- Zadeh L. A. (1965). Fuzzy Sets, *Information and Control*, 8, 338-353.
- Zadeh L. A. (1968). Fuzzy algorithm, *Information and Control*, 12, 94-102.
- Zadeh L. A. (1973). Outline of A New Approach to the Analysis of Complex Systems and Decision Processes. *IEEE Transaction on Systems, Man, and Cybernetics*, 3, 28-44.
- Zadeh L. A. (1975). The Conception of A Linguistic Variable and Its Application in Approximate Reasoning – I. *Information Science*, 8, 199-249.
- Zadeh L. A. (1996). Fuzzy Logic = Computing with Words. *IEEE transactions on fuzzy systems*, 4(2), 103-111.
- Zadeh L. A. (1999). From Computing with Numbers to Computing with Words – From Manipulation of Measurements to Manipulation of Perceptions. *IEEE transactions in circuits and systems*, 45(1), 105-119.
- Zhang G. M., Hwang T. W. & Ratnakar R. (1993). Mathematical modeling of the uncertainty for improving quality in machining operation. *Proceedings Second International Symposium on Uncertainty Modeling and Analysis*, pp. 500-505.